“C” COURSE CATALOG

2020/2021 SCHOOL YEAR

2020/2021

ENCORE EDUCATION CORPORATION

16955 Lemon Street, Hesperia, CA 92345

COURSE CATALOG 2020/2021

Contents

College Board Approved A-G Courses

Mathematics

Algebra 2A and 2B

Basic Course Information:
Title: Algebra 2A and 2B (online)
Length of Course: Full Year
Subject Area: Mathematics (c) / Algebra II
UC Honors Designation? No
Prerequisite: Algebra I
Co-requisites: None
Integrated (Academics / CTE): No
Grade Levels: 11th & 12th
Course Description: This course is adopted from **Cyber High**. Please refer to their course list for a full course description.
In Algebra 2A and 2B, students build on mathematical concepts learned in Algebra and Geometry by extending their knowledge through the study of functions (polynomial, rational, radical, quadratic, exponential, and logarithmic), systems of equations and inequalities, modeling (linear, quadratic, and exponential), trigonometric functions, and probability and statistics. Problems are designed to engage higher order thinking processes in a collaborative environment and provide opportunities for practical applications of the concepts developed within the course. Students work with the concepts in various applications including practice exercises, solving word problems, and working with real-world situations, and they have the opportunity to develop and utilize analytical skills, think critically, develop logical thought processes, and make valid inferences. The plan of instruction includes demonstration, modeling, guided practice, and independent application which will lead students to broaden their scope of the problem-solving process. Activities and performance-based learning tasks are designed to engage higher order thinking processes and provide opportunities for practical applications of the concepts developed within the course.

Algebra A and B

Basic Course Information:

Title: Algebra A and B (online)
Length of Course: Full Year
Subject Area: Mathematics (c) / Algebra I
UC Honors Designation? No
Prerequisite: None
Co-requisites: None
Integrated (Academics / CTE): No
Grade Levels: 9th
Course Description: This course is adopted from Cyber High. Please refer to their course list for a full course description.
In conjunction with Algebra B, this course shows how algebraic skills are applied in a wide variety of problem-solving situations and, in seeing the larger picture and in understanding the underlying concepts, students will be in a better position to apply their knowledge to new situations and problems. Students review Pre-Algebra skills (including variables, expressions, order of operations, and equations) and the fundamentals of the language of mathematics. As students progress through the course, they will study concepts like sequences and their graphs, independent and dependent relationships, how to simplify and solve equations and functions, monomials and polynomials, factorization, exponential graphs and functions, transformations, slope, how to solve systems, square roots, quadratic equations, inequalities, absolute value, statistics, etc. Much of the course covers abstract relationships and their manipulations, but it also involves algebraic thinking and the application of these skills to word problems and real life situations. Problems are designed to engage higher order thinking processes in a collaborative environment and provide opportunities for practical applications of the concepts developed within the course. Students have the opportunity to develop and utilize analytical skills, think critically, develop logical thought processes, and make valid inferences. The plan of instruction includes demonstration, modeling, guided practice, and independence which will lead students to broaden their scope of the problem-solving process. Questions, activities, and performance-based learning tasks are designed to engage higher order thinking processes and provide opportunities for practical applications of the concepts developed within the course.

Calculus Honors

Basic Course Information:
Title: Calculus Honors
Length of Course: Full Year
Subject Area: Mathematics (c) / Calculus
UC Honors Designation? Yes
Prerequisite: Pre Calculus

Co-requisites: None
Integrated (Academics / CTE): No
Grade Levels: 11th & 12th
Course Description: Calculus honors is a one year course designed to meet the California State Standards for calculus. The topics covered are: limits, derivatives, definite & indefinite integrals, and applications of these topics. Subtopics include products, quotients, the calculus of logarithmic functions, growth and decay, plane and solid figures, algebraic calculus techniques, and the calculus of motion. These topics will be explored graphically, numerically, algebraically and verbally. In addition to these topics, this course will equip students with the ability to understand and apply the formal definition and the graphical interpretation of calculus topics. It will provide students with an understanding of how calculus can be applied to everyday situations in business, economics, environmental science, health care, life science, social science, sports, technology, and physical science. Finally, it will introduce students to a field of study that they might pursue further on the university level. Our version of calculus honors is based heavily on AP Calculus so that, even though the course is not considered AP, students who wish to take the exam can do so. It is comparable to a college level Calculus I class.
Unit 1: Pre-requisites for Calculus
Slopes, lines, and linear equations will be reviewed in this unit. Average rate of change and slope of the secant line will lead to instantaneous rate of change and the slope of the tangent line. Difference quotient will be used and then we will progress to the limit of the difference quotient as approaches zero. Point-slope form will be emphasized. The relationship of the slopes of a function and its inverse will be revisited. We will conduct a quick and concise review of trigonometric ratios and basic trigonometric identities. Functions and their graphs will be reviewed including domain and range, odd and even functions, the graph of a semi-circle, the absolute value function, greatest integer function, piecewise functions, exponential functions, logarithmic functions, inverse functions, and composite functions. The difference quotient will be used to find average rate of change of a function. In the next unit, we will explore how the limit of the difference quotient leads to the derivative.
Unit 2: Limits and Continuity
This unit begins with rates of change—average rate of change and instantaneous rate of change. Rates of change will be analyzed by table, by graph, or by equation. Limits will be introduced graphically, algebraically, and numerically. Limits will be defined informally and formally. One-sided limits, when a limit fails to exist, limit theorems, limits involving infinity, and limits involving the difference quotient will all be explored. This is where we make connections as we transition from slope to difference quotient then derivative. Continuity will be defined informally and formally. Continuity will be explored for piecewise functions. Continuity will be analyzed at a hole, at a jump discontinuity, and at vertical asymptotes. Students will also apply the Intermediate Value Theorem and the Extreme Value Theorem to continuous, differentiable functions on a closed interval. ***Unit 2 Graphing Software and Calculator Use:*** Calculators are used to visualize and interpret limits from graphs, equations, and data tables. Whole class discussions, think-pair-share, and small groups (3-4 students) are used to communicate mathematical concepts, compare solutions, and discuss why some limits exist and why others do not.
Unit 3: Derivatives: graphically, algebraically, and numerically
We start with the concept of local linearity in this chapter. Then we learn that differentiability implies continuity. We will transition from the limit of the difference quotient (or the limit of the slope) definition of derivative to some of the more efficient techniques for finding derivatives. We will use point-slope form to find tangent and normal lines. We will use the power rule to find first

derivatives, second derivatives, and higher order derivatives. Application problems in this unit include particle motion involving position, displacement, total distance travelled, velocity, speed and acceleration. Students will be expected to determine if an object is speeding up or slowing down, where a function is increasing or decreasing, and the concavity of a function. Students will analyze functions graphically, algebraically, and numerically. Techniques of differentiation will be explored and practiced to proficiency. These include the power rule, product rule, quotient rule, and the chain rule. Students will discover the six trig derivatives and the six inverse trig derivatives. These rules will be derived, proven, and used to solve variety of problems, both theoretical and applied. Students will learn implicit differentiation and use implicit techniques to solve related rates problems. They will also find derivatives of exponential and logarithmic functions. Students will use curve sketching to analyze functions visually and graphically. They will find exact maxima, minima, and points of inflection. They will use the first derivative test and the second derivative test to find extrema. They will determine when a function is increasing, decreasing, concave up, and concave down. Students will also analyze the behavior of a function at a cusp, corner, vertical asymptote or other discontinuity, and the end behavior of the function. Students will use L'Hopital's Rule to find the limit of a function at a point resulting in indeterminate form. ***Unit 3 Graphing software and Calculator Use:*** Students are introduced to the 2nd-calc-6 feature on their graphing calculator to find dy/dx at a particular value and the nDeriv function to graph derivatives at one or all x-values. The calculator helps facilitate class discussion comparing a function with its first derivative and second derivative, comparing extrema of *f* (x) with zeros of *f* '(x), comparing points of inflection of *f* (x) with extrema of the *f* '(x), comparing the degree of *f* (x) with the degree of *f* '(x), and the overall discussion of increasing and decreasing slope.
Unit 4: Applications of the Derivative
Students will learn to find extreme values on a closed interval, examining candidates at critical points and endpoints. They will apply the Extreme Value Theorem. Students will also apply the Mean Value Theorem and Rolle's Theorem to differentiable functions on a given interval. Students will solve optimization problems and related rates problems. ***Unit 4 Graphing software and Calculator Use:*** Calculator use will be ongoing and regular as students solve application problems in this unit.
Unit 5: Definite and Indefinite Integrals
Students will learn to solve definite and indefinite integrals. First they will be introduced to the Trapezoid Rule and Riemann Sums – left, right, and midpoint. They will learn the Fundamental Theorem of Calculus–Part 1 and Part 2. They will understand the Mean Value Theorem for definite integrals, properties of definite and indefinite integrals. They will learn to solve definite integrals with a negative integrand and definite integrals where the lower bound is greater than the upper bound. Students will find anti-derivatives of powers, trig functions, exponential and logarithmic functions. Students will find anti-derivatives that result in inverse trig functions. Students will become proficient at the techniques of u-substitution, integration by parts, and integration by using partial fractions. U-substitution will be used for both definite and indefinite integrals. Students will complete a Riemann Sum investigation which will include in-depth analysis of a function and the area between the curve and the x-axis. Students will compare the left Riemann sum, the right Riemann sum, the midpoint Riemann sum, the Trapezoid sum, and the definite integral result. Students will find when an approximation overestimates or underestimates the actual area and they will make and justify conclusions about the accuracy of

each sum as an estimate of the area. Students will use anti-differentiation to solve a variety of application problems including accumulation (input-output) problems and particle motion problems. ***Unit 5 Graphing software and Calculator Use:*** Using graphing technology and available programs, we explore Riemann sums and the Trapezoid Rule. Calculators are used to confirm definite integrals that we have first found numerically. We also discuss regions below the x-axis and regions that are found both above and below the x-axis. Students use the 2nd-calc-7 function to find the anti-derivative of a function on a closed interval. Class discussion emphasizes the concept of area under the curve.
Unit 6: The Calculus of Growth and Decay
Slope Fields will be drawn and analyzed in this unit. Separable differential equations will be solved using a four step process (separate the variables, integrate both sided, find the general solution including the constant of integration, solve for the particular solution). Applications will include exponential growth and decay, predator-prey population problems and direct proportion differential equations that lead to the exponential function (where C is the constant of integration and k is the constant of proportion), and other differential equations for real-world applications. Students will learn Euler's Method for solving differential equations in this unit. Unit 6 also includes the technique of u-substitution for solving indefinite and definite integrals. ***Unit 6 Graphing software and Calculator Use:*** Calculator use will be embedded in unit 6. The graphing calculator will be allowed for five primary purposes. 1. to plot the graph of a function, as needed 2. to find x-intercepts or zeros of a function 3. to numerically calculate a derivative 4. to numerically calculate the value of a definite integral 5. for exploration Graphing calculator software may also be used to generate slope fields for analysis.
Unit 7: The Calculus of Plan and Solid Figures
In this unit, students will learn to find the area between two curves. Cross sections may be perpendicular to the x-axis or the y-axis. Some problems will be calculator enabled and some will not. Students will learn to find the volume of a solid of revolution with both horizontal and vertical axes. Students will also find the volume of a solid with known cross-sections. Cross sections may be perpendicular to the x-axis or the y-axis. Students will learn to find volumes of revolution by using the disk method, the washer method, and by cylindrical shells. This unit will also include finding the length of a plane curve, aka finding arc length.
Unit 8: additional Techniques for Integration
Students will use integration by parts to find the integral of a product of two functions. Students will learn the ILATE acronym for choosing the parts in integration by parts. Students will also learn tabular integration, aka rapid repeated integration by parts. In this unit, students will find the integral of the natural logarithm function and the common logarithmic function. Students will also use trigonometric substitution, specifically the power reduction formulas to find integrals of powers of trigonometric functions. Students will also use advanced trigonometric substitution to find integrals of trigonometric functions. Integration of Rational Functions will be solved using the technique of partial fraction decomposition. Partial fractions will include proper and improper fractions in the integrand. Students will also integrate piecewise continuous functions.
Honors Final Exam Details:
A comprehensive - 2 hour - final exam will be given at the end of each semester. The semester 1 final exam will cover units 1-4 and be comprised of multiple choice, short answer and performance

task questions. The final at the end of semester 2 will encompass all 8 units learned in Calculus Honors. The semester 2 final will also consist of multiple choice, short answer and performance task questions. Students will also complete a project each semester. Projects include: instructional videos, power point / brochure, research papers, and computer simulation models.

Integrated Math I

Basic Course Information:
Title: Integrated Math I
Length of Course: Full Year
Subject Area: Mathematics (c) / Mathematics I
UC Honors Designation? No
Prerequisite: None
Co-requisites: None
Integrated (Academics / CTE): No
Grade Levels: 9th

Course Description: Integrated Math 1 is the first course of a three-course sequence including Integrated Math 1, 2, and 3. This course is aligned with the Common Core standards for Integrated Math 1. The content standards for Integrated Math 1 and standards for mathematical practice can be viewed on the CDE website.

In this course, students review and develop skills learned in middle school math course and proceed into higher level mathematical reasoning, teaching them to understand and apply mathematical concepts and tools in the following ways; graphically, numerically, algebraically, and in written and spoken presentations. This course will also show physical and realistic application of mathematics and how mathematics is a great tool for problem solving in many areas of life (from personal finances to workplace applications). Students who are successful in this course will be advanced to Integrated Math 2. There are 8 modules to this class. **Each module consists of the following:**

- **Classwork Task**: Launch – whole class, Explore – individual, pairs or small groups, Discuss – whole class
- **Homework**: assignments have been designed to continue to spiral a review of content.
- **Quizzes**: two quizzes per module consisting of multiple-choice, free response, and extended response items.
- **End of Module Exam**: one exam per module consisting of multiple-choice, free response, and extended response items.

Module 1: Getting Ready

This module introduces students to the task-based curriculum and will review solving linear equations. The module then extends to solving inequalities. The tasks covered are:

Defining quantities and interpreting expressions

Interpreting expressions and using units to understand problems

Using units as a way to understand problems

Explaining each step in the process of solving an equation

Rearranging formulas to solve for a variable

Solving literal equations

Writing inequalities to fit a context

Reasoning about inequalities and the properties of inequalities

Solving linear inequalities and representing the solution

Task: Checkerboard Borders

The focus of this task is on the generation of multiple expressions that connect with the visuals provided. These expressions will also provide opportunity to discuss equivalent expressions and review the skills students have previously learned about simplifying expressions using variables.

Students will learn:

-how to write expressions to model a situation

-how to use dimensional analysis

-solve equations (including literal equations) & inequalities

Module 2: Systems of Equations & Inequalities

Overview: In this module, students will solve systems of linear functions using methods which include substitution, graphing, and linear combinations. Students will extend what they have learned about systems of equations to write and solve systems of inequalities with up to four constraints. They will end by learning to find a profit. The tasks covered are:

Represent constraints with systems of inequalities.

Write and graph linear inequalities in two variables.

Write and solve equations in two variables.

Graph the solution set to a linear system of inequalities.

Solve systems of linear equations in two variables.

Solve systems of linear equations by elimination.

Solve systems of linear inequalities representing constraints.

Work with systems of linear equations, including inconsistent and dependent systems.

Work with systems of linear inequalities and their boundaries.

Use systems of linear equations and inequalities in a modeling context.

Task – Pet Sitters.

As students work with the context of making recommendations for how many dogs and cats the pet sitters should plan to accommodate, they will encounter many ideas, strategies and representations related to solving systems of equations and inequalities. For example, they will explore the notion of constraints since in this task the number of each type of pet that can be accommodated is limited by space and money, but many different combinations of dogs and cats are possible. They may consider the notion of a system of equations since each constraint (space, start-up costs) allows for a different set of possibilities—a particular combination of dogs and cats may satisfy one constraint but not another—so both constraints must be considered simultaneously. Finally, they may surface the notion of a system of inequalities since the pet sitters don't have to use up all of the available space or money, implying that each constraint may be represented by an inequality. Module 2: The students will learn: -how to write expressions to model a situation -how to use dimensional analysis -solve equations (including literal equations) & inequalities -graph linear equations *Students evaluate which linear equation forms are best suited for various scenarios. *Students verify that a point lies on a line given an equation of the line. Students are able to derive linear equations by using the point-slope format.
Module 3 – Arithmetic and Geometric Sequences
Overview: This module introduces students to sequences, and then focuses student attention on arithmetic and geometric sequences. Students then use recursive and explicit formulas to determine subsequent terms of a sequence. The relationship between arithmetic sequences and linear functions and some geometric sequences and exponential functions is developed. The tasks covered are: Represent arithmetic sequences with equations, tables, graphs, and story context. Represent geometric sequences with equations, tables, graphs, and story context Arithmetic Sequences: Constant difference between consecutive terms. Geometric Sequences: Constant ratio between consecutive terms. Arithmetic Sequences: Increasing and decreasing at a constant rate. Compare rates of growth in arithmetic and geometric sequences. Recursive and explicit equations for arithmetic and geometric sequences. Use rate of change to find missing terms in an arithmetic sequence. Use a constant ratio to find missing terms in a geometric sequence. Develop fluency with geometric and arithmetic sequences. **Task** – Chew On This.

The purpose of this task is to solidify and extend the idea that geometric sequences have a constant ratio between consecutive terms to include sequences that are decreasing ($0 < r < 1$). The common ratio in one geometric sequence is a whole number and in the other sequence it is a percent. This task contains an opportunity to compare the growth of arithmetic and geometric sequences. This task also provides practice in writing and using formulas for arithmetic sequences.

Students will learn:

-how to read, write arithmetic & geometric sequences & distinguish between them,

-model with and represent arithmetic & geometric sequences in tables or graphs

-write explicit & recursive functions for arithmetic & geometric sequences

Module 4 – Linear & Exponential Functions

Overview: This module continues the work with arithmetic and geometric sequences as a way of comparing and contrasting linear and exponential functions. Students explore the graphical behavior of exponential functions including intercepts, asymptotes, domain and range, and interval of increase and decrease. Students understand how the common ratio in an exponential function determines whether the function is increasing or decreasing. The tasks covered are:

Introduce continuous linear and exponential functions.

Define linear and exponential functions based upon the pattern of change.

Identify rates of change in linear and exponential functions.

Distinguish between linear and exponential functions using various representations.

Compare the growth of linear and exponential functions.

Compare linear and exponential models of population.

Interpret equations that model linear and exponential functions.

Evaluate the use of various forms of linear and exponential equations.

Understand and interpret formulas for exponential growth and decay.

Solve exponential and linear equations.

Task – Growing, Growing, Gone.

The purpose of this task is for students to use their understanding of linear and exponential patterns of growth to model the growth of a population. Students are given two data points and asked to create both an exponential and a linear model containing the points. Students may draw upon their experience with arithmetic and geometric means to develop new points in the model. The task provides opportunities to create tables, equations, and graphs and use those representations to argue which model is the best fit for the data.

Students will learn:

-how to recognize linear & exponential situations

-how to write functions (linear or exponential) in explicit and recursive formats

-convert between representations for linear and exponential functions (graphs, tables, functions, written descriptions of scenarios)

-compare rates of change such as slope, instantaneous rates of change of curves, and absolute vs. relative rates of change.

-Students model various real life situations with linear and exponential models.

Module 5: Features of Functions

Overview: In this module students extend their knowledge of functions learned previously by exploring the concept of independent and dependent quantities, understanding the concept of domain, analyzing functions based on their characteristics, and writing and graphing functions given a real-world scenario. Students will also write algebraic representations of functions and compare them to other representations such as tables, graphs, and verbal descriptions. Students understand that there are many ways to represent a function: algebraic, graphic, verbal, and a table, and they will learn to produce any representation given one of the others. The tasks covered are:

Use a story context to graph and describe key features of functions.

Use tables and graphs to interpret key features of functions.

Features of functions using various representations.

Interpret functions using notation.

Combine functions and analyze contexts using functions.

Use graphs to solve problems given in function notation.

Define function.

Identify whether or not a relation is a function given various representations.

Match features and representations of a specific function.

Task – The Water Park.

The purpose of this task is for students to interpret and highlight features of functions using contexts. This task provides opportunities for students to practice skills they have already learned as well as solidifying their knowledge of features of functions. This task first asks students to make observations from a graph. There are several observations to make and by having students make these observations, they are accessing their background knowledge as a way to prepare for this task. In the following sections, students solidify their understanding of domain and distinguish between the domain of a function and the domain of a situation. They also use function notation to interpret the meaning of the situation. The following mathematics should be addressed in this task:

- Interpreting x- and y-intercepts
- Comparing rates of change
- Finding where two functions are equivalent
- Connecting the equation to a graph and appropriately labeling a graph
- Determine the domain of a function as well as the restricted domain due to a story context

- Interpreting function notation for both input and output values

Students will learn:

-how to read, write & graph functions

-to distinguish between functions & relations

-how to describe features of functions

-convert between representations of functions

-use academic vocabulary for functions; domain, range, etc.

-evaluate functions when given a graph or the function

-solve linear systems as functions

-Operate on functions, such as f(1)-g(1), etc.

Module 6 – Congruence, Construction & Proof

Overview: This module focuses on rigid motions that preserve congruency and basic geometric constructions. Students will learn that rotations, reflections, and translations all preserve congruency. Students will develop the definition of congruency based on rigid motions and use this definition to determine under what circumstances triangles are congruent. Finally, students learn to show triangles are congruent using the SSS, SAS, and ASA congruence theorems. Students will perform constructions using a compass and straightedge. The tasks covered are:

Develop the definitions of the rigid-motion transformations: translations, reflections and rotations.

Examine the slope of perpendicular lines.

Determine which rigid-motion transformation carry one image onto another congruent image.

Write and apply formal definitions of the rigid-motion transformations: translations, reflections and rotations.

Find rotational symmetry and lines of symmetry in special types of quadrilaterals.

Examine characteristics of regular polygons that emerge from rotational symmetry and lines of symmetry.

Make and justify properties of quadrilaterals using symmetry transformations.

Describe a sequence of transformations that will carry congruent images onto each other.

Establish the ASA, SAS and SSS criteria for congruent triangles.

Work with systems of linear equations, including inconsistent and dependent systems.

Explore compass and straightedge constructions to construct rhombuses and squares.

Explore compass and straightedge construction to construct parallelograms, equilateral triangles and inscribed hexagons.

Examine why compass and straightedge constructions produce the desired objects.

Write procedures for compass and straightedge constructions.

Task – Leaping Lizards.

This task provides an opportunity for formative assessment of what students already know about the three rigid-motion transformations: translations, reflections, and rotations. As students engage in the task they should recognize a need for precise definitions of each of these transformations so that the final image under each transformation is a unique figure, rather than an ill-defined sketch. In addition to the work with the rigid-motion transformations, this task also surfaces thinking about the slope criteria for determining when lines are parallel or perpendicular. Finally, this task reminds students that rigid-motion transformations preserves distance and angle measures—implying that the figures forming the pre-image and image are congruent. Students will be attending to two different categories of distances—the lengths of line segments that are used in the definitions of the transformations, and the lengths of the congruent line segments that are contained within the pre-image and image figures themselves. Students may determine that these lengths are preserved by counting units of "rise" and "run", or by using the Pythagorean Theorem. Ultimately, this work will lead to the development of the distance formula in future tasks.

Students will learn:

-how to perform transformations of figures in a coordinate plane (translations, reflections, rotations & dilations)

-how to use the Pythagorean Theorem to find missing lengths of right triangles

-how to find the slope of a line that is parallel or perpendicular to a given line or segment

-use academic vocabulary for transformations such as pre-image, prime markings, composition, mapping, etc. and plane figures (parallelograms, rhombi, etc.)

-analyze & describe symmetries

-calculate various angles for polygons such as interior & exterior angles for regular polygons

-define & distinguish between congruence & similarity when & how to apply triangle congruence rules (SSS, ASA, etc.)

-how to write triangle congruence proofs

-basic constructions & how to combine them to form congruent triangles

-how to use proofs to justify basic constructions

Module 7: Connecting Algebra & Geometry

Overview: This module makes connections between geometry and algebra. Students will derive the midpoint and distance formulas using the coordinate plane and the Pythagorean Theorem. Students will develop the rules for slopes of parallel and perpendicular lines and use these rules to understand how to write the equations of parallel and perpendicular lines and calculate perimeter and area of various geometric figures represented on the coordinate plane. Students learn that a translation preserves both angle and distance. The tasks covered are:

Use coordinates to find distances and determine the perimeter of geometric shapes.

Prove slope criteria for parallel and perpendicular lines.

Use coordinates to algebraically prove geometric theorems.

Write the equation f(t) = m(t) + k by comparing parallel lines an finding k.

Determine the transformation from one function to another.

Translate linear and exponential functions using multiple representations.

Task – Training Day.

Students have had a lot of experience with linear functions and their relationships. They have also become more comfortable with function notation and features of functions. In this task, students first make observations about the rate of change and the distance traveled by the two runners. Using their background knowledge of linear functions, students start to surface ideas about vertical translations of functions and how to build one function from another.

Task – Training Day Part II.

Students will solidify their understanding of vertical transformations of linear functions in this task.

Goals of this task include:

- Writing function transformations using function notation.
- Recognizing that the general form y = f(x)+ k represents a vertical translation, with the output values changing while the input values stay the same.
- Understanding that a vertical shift of a linear function results in a line parallel to the original.

Students will learn:

-how to use coordinates to find distances and calculate perimeter of plane figures

-how to determine & describe how one function is transformed into another

-translate linear & exponential functions in a variety of representations

-use constructions & algebra to validate rules about parallel & perpendicular lines

-graph parallel & perpendicular lines from a variety of description forms (written, function notation, transformation notation, etc.)

-classify polygons based on their attributes & properties

-perform transformations on functions

Module 8: Modeling Data

Overview: This module reviews data analysis of data sets with one variable. Students review how to represent data graphically through dot plots, histograms, and box-and-whisker plots. The module leads students to determine measures of center for a data set, determine outliers in a data set, and determine the interquartile range and standard deviation for data sets. This module will also introduce students to bivariate data. Students will learn to use linear regression and determine the association of the data. Students will be able to compute and interpret the correlation coefficient of the data set. Students will be introduced to residuals and learn to create residual plots using graphing calculators or GeoGebra. Students will learn how to create and analyze a two-way table by looking at conditional relative frequency. The tasks covered are:

Use context to describe data distributions and compare statistical representations.

Describe data distributions and compare two or more data sets.

Interpret two way frequency tables.

Use context to interpret and write conditional statements using relative frequency tables.

Develop an understanding of the value of the correlation coefficient.

Estimate correlation and lines of best fit. Compare to the calculated results of linear regressions and the correlation coefficient.

Use linear models of data and interpret the slope and intercept of regression lines with various units.

Use residual plots to analyze the strength of a linear model for data.

Task – Making More $.

The purpose of this task is to solidify understanding of correlation coefficient and to develop linear models for data. Students are asked to estimate and calculate correlation coefficients. In the task they estimate lines of best fit and then compare them to the calculated linear regression. The task demonstrates the dangers of using a linear model to extrapolate well beyond the actual data. The task ends with an opportunity to use the correlation coefficient and scatter plot to determine the appropriateness of a linear model.

Students will learn

-how to use context to describe data distribution and compare statistical representations

-how to describe data distributions and compare two or more data sets

-how to interpret two way frequency tables

-how to use context to interpret and write conditional statements using relative frequency tables

-an understanding of the value of the correlation co-efficient

-how to estimate correlation and lines of best fit.

-how to compare to the calculated results of linear regressions and correlation co-efficients

-how to use linear models of data and interpret the slope and intercept of regression lines with various units

-how to use residual plots to analyze the strength of a linear model for data

-Students find lines of best fit, determine correlations and use these to make predictions.

Integrated Math II

Basic Course Information:
Title: Integrated Math II
Length of Course: Full Year
Subject Area: Mathematics (c) / Mathematics II
UC Honors Designation? No
Prerequisite: Math I
Co-requisites: None
Integrated (Academics / CTE): No
Grade Levels: 10th

Course Description: Integrated Math II is the second course of a three-course sequence including Integrated Math I, II, and III. This course is aligned with the Common Core standards for Integrated Math II. The content standards for Integrated Math II and standards for mathematical practice can be viewed on the CDE website.

This course will reinforce concepts and skills from Integrated Math I and will prepare students for Integrated Math III. For the high school Model Mathematics II course, there are five critical areas: (1) extend the laws of exponents to rational exponents; (2) compare key characteristics of quadratic functions with those of linear and exponential functions; (3) create and solve equations and inequalities involving linear, exponential, and quadratic expressions; (4) extend work with probability; and (5) establish criteria for similarity of triangles based on dilations and proportional reasoning.

There are 9 modules to this class. **Each module consists of the following:**

- **Classwork Task**: Launch – whole class, Explore – individual, pairs or small groups, Discuss – whole class
- **Homework**: assignments have been designed to continue to spiral a review of content.
- **Quizzes**: two quizzes per module consisting of multiple-choice, free response, and extended response items.
- **End of Module Exam**: one exam per module consisting of multiple-choice, free response, and extended response items.

Module 1: Quadratic Functions

Overview: This module introduces students to quadratic functions. The students will classify functions as linear, exponential or quadratic. They will analyze first & second differences & use them to classify functions. They'll write explicit functions and write recursive functions. Students will also use features of quadratics to solve real-world problems such as the maximum or minimum value of a height of a projectile, a profit made or an optimum area.

The tasks covered are:

- An introduction to quadratic functions, designed to elicit representations and surface a new type of pattern and change.
- Solidification of quadratic functions begins as quadratic patterns are examined in multiple representations and contrasted with linear relationships.
- Focus specifically on the nature of change between values in a quadratic being linear.

- Focus on maximum/minimum point as well as domain and range for quadratics.
- Examining quadratic functions on various sized intervals to determine average rates of change.
- Comparing quadratic and exponential functions to clarify and distinguish between type of growth in each as well as how that growth appears in each of their representations
- Incorporating quadratics with the understandings of linear and exponential functions

Task: Rabbit Run, A Solidify Understanding Task

Misha has a new rabbit that she named "Wascal". She wants to build Wascal a pen so that the rabbit has space to move around safely. Misha has purchased a 72 foot roll of fencing to build a rectangular pen.

1. If Misha uses the whole roll of fencing, what are some of the possible dimensions of the pen?
2. If Misha wants a pen with the largest possible area, what dimensions should she use for the sides? Justify your answer.
3. Write a model for the area of the rectangular pen in terms of the length of one side. Include both an equation and a graph.
4. What kind of function is this? Why?

Core Standards Focus:

- BF.1 Write a function that describes a relationship between two quantities. *
- Determine an explicit expression, a recursive process, or steps for calculation from a context.
- SSE.1 Interpret expressions that represent a quantity in terms of its context.*
- Interpret parts of an expression, such as terms factors, and coefficients.
- CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales help.

Students will learn how to: The purpose of this task is to solidify and extend student thinking about quadratic functions to include those with a maximum point. Students will use the graph of the function to discuss the domain and range of a continuous quadratic function in addition to identifying the maximum value and finding the intervals on which the function is increasing and decreasing.

Module 2: Structures of Expressions

Overview: In this module, students will learn to graph quadratics using patterns (first & second differences), vertices, tables, and functions in standard form, vertex form and factored form. Students will convert forms of quadratic functions between standard form, vertex form and factored form. They will use a convenient form to find intercepts, vertices, and then make tables and graphs. They will factor quadratics. Finally, they will complete squares, using both algebraic and physical methods. The tasks covered are:

- Connecting transformations to quadratic functions and parabolas
- Working with vertex form of a quadratic, connecting the components to transformations
- Visual and algebraic approaches to completing the square
- Connecting the factored and expanded or standard forms of a quadratic

- Focus on the vertex and intercepts for quadratics
- Building fluency in rewriting and connecting different forms of a quadratic

<u>Task: DEVELOPING A PERFECT SQUARE</u>

Part 1: Quadratic Quilts

Optima has a quilt shop where she sells many colorful quilt blocks for people who want to make their own quilts. She has quilt designs that are made so that the can be sized to fit any bed. She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square x, and the area of the basic square is the function . In this way, she can customize the designs by making bigger squares or smaller squares.

What patterns do you notice when you relate the diagrams to the two expressions for the area?

Optima likes to have her little dog, Clementine, around the shop. One day the dog got a little hungry and started to chew up the orders. When Optima found the orders, one of them was so chewed up that there were only partial expressions for the area remaining. Help Optima by completing each of the following expressions for the area so that they describe a perfect square. Then, write the two equivalent equations for the area of the square.

Will the strategy work if b is an odd number? What happens to c if b is odd?

Sometimes a customer orders more than one quilt block of a given size. For instance, when a customer orders 4 blocks of the basic size, the customer service representatives write up an order for A(�) = 4�2.

What would they write if the order was for 2 blocks that are 1 inch longer than the basic block? Write the area function in two equivalent forms. Verify your algebra using a diagram.

Part 2: Quilts and Quadratic Graphs

Optima's niece, Jenny works in the shop, taking orders and drawing quilt diagrams. When the shop isn't too busy, Jenny pulls out her math homework and works on it. One day, she is working on graphing parabolas and notices that the equations she is working with look a lot like an order for a quilt block. For instance, Jenny is supposed to graph the equation: � = (� − 3)2 + 4 . She thinks, That's funny. This would be an order where the length of the standard square is reduced by 3 and then we add a little piece of fabric that has as area of 4. We don't usually get orders like that, but it still makes sense. I better get back to thinking about parabolas. Hmmm…

7. Fully describe the parabola that Jenny has been assigned to graph.
8. Jenny returns to her homework, which is about graphing quadratic functions. Much to her dismay, she finds that she has been given: � = �2 − 6� + 9. Oh dear , thinks Jenny. I can't tell where the vertex is or any of the transformations of the parabola in this form. Now what am I supposed to do? Wait a minute—is this the area of a perfect square? Use your work from Part 1of this task to answer Jenny's question and justify your answer.
9. Jenny says, I think I've figured out how to change the form of my quadratic equation so that I can graph the parabola. I'll check to see if I can make my equation a perfect square. Jenny's equation is: � = �2 − 6� + 9. Change the form of the equation, find the vertex, and graph the parabola.

10. The next quadratic to graph on Jenny’s homework is � = �2 + 4� + 2. Does this expression fit the pattern for a perfect square? Why or why not?

Core Standards Focus:

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

1. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Students will learn: The purpose of this task is to extend students’ understanding of graphing quadratic functions to include equations written in standard form. In the task, students will use diagrams of area models to make sense of the terms in a perfect square trinomial and discover the relationship between coefficients of a quadratic equation in standard form. They will use this understanding to complete the square to find an equivalent form of an equation. Students will also use completing the square to find the vertex form of a quadratic function and graph the associated parabola.

Module 3: Quadratic Equations

Overview: In this module, students will learn how to connect radicals and rules of exponents to create meaning for rational exponents. They will verify that properties of exponents hold true for rational exponents. Students will convert between exponential and radical forms of expressions. They will develop the Quadratic Formula as a way for finding x-intercepts and roots of quadratic functions. Students will evaluate how different forms of a quadratic expression can facilitate the solving of quadratic equations. They will solve various forms of quadratic equations and perform operations with real and complex numbers

Task: CURBSIDE RIVALRY

Carlos and Clarita have a brilliant idea for how they will earn money this summer. Since the community in which they live includes many high schools, a couple of universities, and even some professional sports teams, it seems that everyone has a favorite team they like to root for. In Carlos’ and Clarita’s neighborhood these rivalries take on special meaning, since many of the neighbors support different teams. They have observed that their neighbors often display handmade posters and other items to make their support of their favorite team known. The twins believe they can get people in the neighborhood to buy into their new project: painting team logos on curbs or driveways.

For a small fee, Carlos and Clarita will paint the logo of a team on a neighbor’s curb, next to their house number. For a larger fee, the twins will paint a mascot on the driveway. Carlos and Clarita have designed stencils to make the painting easier and they have priced the cost of supplies. They have also surveyed neighbors to get a sense of how many people in the community might be interested in purchasing their service. Here is what they have decided, based on their research.

- A curbside logo will require 48 in2 of paint
- A driveway mascot will require 16 ft2 of paint
- Surveys show the twins can sell 100 driveway mascots at a cost of $20, and they will sell 10

fewer mascots for each additional $5 they charge

1. If a curbside logo is designed in the shape of a square, what will its dimensions be?

A square logo will not fit nicely on a curb, so Carlos and Clarita are experimenting with different types of rectangles. They are using a software application that allows them to stretch or shrink their logo designs to fit different rectangular dimensions.

2. Carlos likes the look of the logo when the rectangle in which it fits is 8 inches longer than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write a quadratic equation that represents these requirements.

3. Clarita prefers the look of the logo when the rectangle in which it fits is 13 inches longer

Than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write a quadratic equation that represents these requirements.

Your quadratic equations on the previous two problems probably started out looking like this: $x(x + n) = 48$ where n represents the number of inches the rectangle is longer than it is wide. The expression on the left of the equation could be multiplied out to get and equation of the form $x^2 + nx = 48$. If we subtract 48 from both sides of this equation we get $x^2 + nx - 48 = 0$. In this form, the expression on the left looks more like the quadratic functions you have been working with in previous tasks, $y = x\,2 + nx - 48$

4. Consider Carlos' quadratic equation where n = 8, so $x^2 + 8x - 48 = 0$. How can we use our work with quadratic functions like $y = x^2 + 8x - 48$ to help us solve the quadratic equation $x^2 + 8x - 48 = 0$? Describe at least two different strategies you might use, and then carry them out. Your strategies should give you solutions to the equation as well as a solution to the question Carlos is trying to answer in #2.

5. Now consider Clarita's quadratic equation where n = 13, so $x^2 + 13x - 48 = 0$. Describe at least two different strategies you might use to solve this equation, and then carry them out. Your strategies should give you solutions to the equation as well as a solution to the question Clarita is trying to answer in #3.

6. After much disagreement, Carlos and Clarita agree to design the curbside logo to fit in a rectangle that is 6 inches longer than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write and solve a quadratic equation that represents these requirements.

7. What are the dimensions of a driveway mascot if it is designed to fit in a rectangle that is 6 feet longer than it is wide? (See the requirements for a driveway mascot given in the bulleted list above.) As part of your work, write and solve a quadratic equation that represents these requirements.

8. What are the dimensions of a driveway mascot if it is designed to fit in a rectangle that is 10 feet longer than it is wide? (See the requirements for a driveway mascot given in the bulleted list above.) As part of your work, write and solve a quadratic equation that represents these requirements.

Carlos and Clarita are also examining the results of their neighborhood survey, trying to determine how much they should charge for a driveway mascot. Remember, this is what they have found from the survey: They can sell 100 driveway mascots at a cost of $20, and they will sell 10 fewer mascots for each additional $5 they charge.

9. Write an equation, make a table, and sketch a graph for the number of driveway mascots the twins can sell for each $5 increment, x, in the price of the mascot.

10. Write an equation, make a table, and sketch a graph (on the same set of axes) for the price of the driveway mascot for each $5 increment, x, in the price.

11. Write an equation, make a table, and sketch a graph for the revenue the twins will collect for each $5 increment in the price of the mascot.

12. The twins estimate that the cost of supplies will be $250 and they would like to make $2000 in profit from selling driveway mascots. Therefore, they will need to collect $2250 in revenue. Write and solve a quadratic equation that represents collecting $2250 in revenue. Be sure to clearly show your strategy for solving this quadratic equation. **Students will learn:** In this task students use their techniques for changing the forms of quadratic expressions (i.e., factoring, completing the square to put the quadratic in vertex form, or using the quadratic formula to find the x-intercepts) as strategies for solving quadratic equations. Core*Standards*Focus: A.REI.4 Solve quadratic equations in one variable. 1. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form. 2. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. Note*for*Mathematics*II*A.REI.4a,*A.REI.4b* Extend to solving any quadratic equation with real coefficients, including those with complex solutions.* A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. Note*for*Mathematics*II*A.REI.7* Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.* Related*Standards:**A.SSE.1*
Module 4: More Functions, More Features
Overview: In this module, students will learn how to evaluate & graph piecewise functions. They will write piecewise functions from a variety of starting points (written description of an abstract or

real-world situation, a table, or a graph.) They will also write, graph and evaluate functions: absolute value functions, piecewise functions and inverse functions. The tasks covered are:

- Build on work from Secondary One to develop understanding of piece-wise functions
- Solidifying understanding of piece-wise functions
- Incorporating absolute value as piece-wise defined functions
- Fluency with absolute value functions and greater understanding of domain and range
- Develop understanding of Inverse functions
- Solidifying inverse functions, what are they, and where they come from
- Using knowledge of features of functions to identify features and to create functions given features

Task: BIKE LOVERS

Michelle and Rashid love going on long bike rides. Every Saturday, they have a particular route they bike together that takes four hours. Below is a piecewise function that estimates the distance they travel for each hour of their bike ride.

1. What part of the bike ride do they go the fastest? Slowest?
2. What is the domain of this function?
3. Find f(2). Explain what this means in terms of the context.
4. How far have they traveled at 3 hours? Write the answer using function notation.
5. What is the total distance they travel on this bike ride?
6. Sketch a graph of the bike ride as a function of distance traveled over time.

Rashid also has a route he likes to do on his own and has the following continuous piecewise function to represent the average distance he travels in minutes:

7. What is the domain for this function? What does the domain tell us?
8. What is the average rate of change during the interval [20, 50]?
9. Over which time interval is the greatest average rate of change?
10. Find the value of each, then complete each sentence frame:
11. *f(30) = __________.* This means…
12. *f(64)= ____________.* This means…
13. *f(10) =* ___________. When finding output values for given input values in a piecewise function, you must …
14. Find the value of *a*
15. Find the value of *b*
16. Sketch a graph of the bike ride as a function of distance traveled as a function of time.

Students will learn: The purpose of this task is for students to solidify their understanding of piecewise functions using their background knowledge of domain and linear functions. Students will solidify their understanding of piece-wise functions by answering questions relating to a piece-wise function using their knowledge of domain to graph each section of the piecewise function graphing complete piecewise-defined functions from equations interpreting the context of a piecewise function

Core Standards Focus:

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain of the function.

F.IF.7b Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

1. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Related Standards: F.IF.4, F.IF.6, F.IF.8

Module 5: Geometric Figures

Overview: In this module, students will learn to write proofs in informal and formal formats, including flow proofs. They will learn to problem solve with angles & triangles, parallel lines. Students will learn to make and test conjectures regarding geometric statements and statements from other settings. They will interpret Venn diagrams and convert to and from conditional statements. Students will use properties of parallelograms and classify quadrilaterals based on properties. Finally, they will write proofs about the centers of triangles. The tasks covered are:

- An introduction to proof illustrated by the triangle interior angle sum theorem
- Reasoning from a diagram to develop proof-like arguments about lines and angles, triangles and parallelograms
- Organizing proofs about lines, angles and triangles using flow diagrams and two column proof formats
- Examining parallelism from a transformational perspective
- Generating conjectures from a diagram and writing formal proofs to prove the conjectures about lines, angles and triangles
- Proving conjectures about parallelograms
- Identifying parallelograms from information about the diagonals
- Reading and writing proofs about the concurrency of medians, angle bisectors and perpendicular bisectors of the sides of a triangle

Task: Parallelism Preserved and Protected

In a previous task, How Do You Know That, you were asked to explain how you knew that this figu re,

Which
was formed by rotating a triangle about the midpoint of one of its sides, was a parallelogram.

You may have found it difficult to explain how you knew that sides of the original triangle and its rotated image were parallel to each other except to say, It just has to be so. There are always some statements we have to accept as true in order to convince ourselves that other things are true.
We try to keep this list of statements as small as possible, and as intuitively obvious as possible. For
example, in our work with transformations we have agreed that distance and angle measures are preserved by rigid motion transformations since our experience with these transformations suggest that sliding, flipping and turning figures do not distort the images in any way. Likewise, parallelism within a figure is preserved by rigid motion transformations: for example, if we reflect a parallelogram the image is still a parallelogram—the opposite sides of the new quadrilateral are still parallel.

Mathematicians call statements that we accept as true without proof postulates. Statements that are supported by justification and proof are called theorems.

Knowing that lines or line segments in a diagram are parallel is often a good place from which to start a chain of reasoning. Almost all descriptions of geometry include a parallel postulate among the list of statements that are accepted as true. In this task we develop some parallel postulates f or rigid motion transformations.

Translations

Under what conditions are the corresponding line segments in an image and its pre-image parallel after a translation? That is, which word best completes this statement?

After a translation, corresponding line segments in an image and its preimage are [never, sometimes, always] parallel.

Give reasons for your answer. If you choose sometimes
, be very clear in your explanation how to
tell when the corresponding line segments before and after the translation are parallel and when they are not.

Rotations

Under what conditions are the corresponding line segments in an image and its pre-image parallel after a rotation? That is, which word best completes this statement?

After a rotation, corresponding line segments in an image and its preimage are [never, sometimes, always] parallel.

Give reasons for your answer. If you choose sometimes , be very clear in your explanation how to tell when the corresponding line segments before and after the rotation are parallel and when they are not.

Reflections

Under what conditions are the corresponding line segments in an image and its pre-image parallel after a reflection? That is, which word best completes this statement?

After a reflection, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.

Give reasons for your answer. If you choose sometimes be very clear in your explanation how to tell when the corresponding line segments before and after the reflection are parallel and when they are not.

Students will
Learn: Euclid was right, we can't make much progress in proving statements in geometry without a statement about parallelism. Euclid made an assumption related to parallelism—his frequently discussed and questioned 5th postulate. Non-Euclidean geometries resulted from mathematicians making different assumptions about parallelism. The purpose of this task is to establish some parallel postulates for transformational geometry. The authors of CCSS-M suggested some statements about parallelism that they would allow us to assume to be true in the ir development of the geometry standards: (1) rigid motion transformations take parallel lines to parallel lines (that is, parallelism, along with distance and angle measure, is preserved by rigid motion transformations—see 8.G.1), and (2) dilations take a line not passing through the center of the dilation to a parallel line (see G.SRT.1a). In this task we develop some additional statements about parallelism for the rigid motion transformations, which we will accept as postulates for our development of geometry: (1) After a translation, corresponding line segments in an image and its preimage are always parallel or lie along the same line; (2) After a rotation of 180°, corresponding line segments in an image and its pre-image are parallel or lie on the same line; (3) After a reflection, line segments in the pre-mage that are parallel to the line of reflection will be parallel to the corresponding line segments in the image.

These statements about parallelism will lead to the proofs of theorems about relationships of angles relative to parallel lines crossed by a transversal.

Note 1: In transformational geometry, one can take the perspective that an image and its pre-image are distinct figures even when they coincide. Consequently, rotating a line 180° about a point on the line creates an image/pre-image pair of lines that coincide. If we consider the image/pre-image lines as distinct, we might also say that they are parallel to each other. Otherwise, they share all points in common and are the same line. In the wording we have used here for translations and rotations, we are taking the perspective that they share all points in common, and therefore, are the same line. **Note 2**: These statements about parallelism could be treated as the orems, rather than postulates,if you wish to pursue more formal proofs about these statements. F

or example, statement 2 about line segments undergoing a 180° rotation being parallel to each other can be proved by contradiction—assume the lines aren't parallel and show that any assumed point of intersection would contradict the assumption that the line had been rotated 180° since the line segment connecting the point of intersection to the center of rotation and back to the point of intersection do es not represent a 180° turn. Such reasoning may be beyond your students, and proof by contradiction is not one of the expected proof formats of the common core standards. **Core Standards Focus**: G.CO.9 Prove theorems about lines and angles. Theorems include: when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent Mathematics II Note for G.CO.10: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while explorin g a variety of formats for expressing that reasoning. Related Standards: 8.G.1, G.SRT.1a
Module 6: Similarity and Right Triangle Trigonometry
Overview: In this module, students will learn to describe, in detail, the effects of dilations on figures, the similarity & proportional properties created by dilations, and relationships of segments intersected by parallel lines. They will prove the Pythagorean Theorem using similar triangles. Students will use similar triangles to build an understanding of trigonometric ratios. They'll find relationships between trigonometric ratios such as the Pythagorean identity. Finally, students will use trigonometric ratios to find unknown lengths and problem solve in real life contexts. The tasks covered are: • Describing the essential features of a dilation • Examining proportionality relationships in triangles that are known to be similar to each other based on dilations • Comparing definitions of similarity based on dilations and relationships between corresponding sides and angles • Examining proportional relationships of segments when two transversals intersect sets of parallel lines • Applying theorems about lines, angles and proportional relationships when parallel lines are crossed by multiple transversals • Applying understanding of similar and congruent triangles to find midpoint or any point on a line segment that partitions the segment in a given ratio • Using similar triangles to prove the Pythagorean theorem and theorems about geometric means in right triangles • Developing and understanding of right triangle trigonometric relationships based on similar triangles • Finding relationships between the sine and cosine ratios for right triangles, including the Pythagorean identity • Solving for unknown values in right triangles using trigonometric ratios

- Practicing setting up and solving right triangles to model real world contexts

Task: 6.2 Triangle Dilations

1. Given ∆ABC, use point M as the center of a dilation to locate the vertices of a triangle that has side lengths that are three times longer than the sides of ∆ABC.
2. Now use point N as the center of a dilation to locate the vertices of a triangle that has side lengths that are one half the length of the sides of ∆ABC.
3. Label the vertices in the two triangles you created in the diagram above. Based on this diagram, write several proportionality statements you believe are true. First write your proportionality statements using the names of the sides of the triangles in your ratios. Then verify that the proportions are true by replacing the side names with their measurements, measured to the nearest millimeter.

My list of proportions: (try to find at least 10 proportionality statements you believe are true)

Students will Learn:

One purpose of this task is to solidify and formalize the definition of dilation: A dilation is a transformation of the plane, such that if O is the center of the dilation and a nonKzero number k is the scale factor, then P' is the image of point P if O, P and P' are collinear and OP'OP = k

A second purpose of this task is to examine proportionality relationships between sides of similar figures by identifying and writing proportionality statements based on corresponding sides of the similar figures.

A third purpose is to examine a similarity theorem that can be proved using dilation: a line parallel to one side of a triangle divides the other two proportionally.

Core Standards Focus:

G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally. Related Standards: G.SRT.1, 8.G.4

Module 7: Circles from a Geometric Perspective

Overview: In this module, students will learn to find the center of rotation using perpendicular bisectors as a tool. They will prove that all circles are similar. They will use relationships between central angles, inscribed angles, circumscribed angles and their arcs. Students will develop & use formulas for perimeter & area of regular polygons. They will justify formulas for circumference and area of circles. They will use proportional reasoning to calculate arc length and sector area. Students will learn to use radians as a measure of angles. They will convert between radians and degrees and use relationships between similar figures to find volumes. The tasks covered are:

- Searching for center of rotation using perpendicular bisectors as a tool

- Proving circles similar
- Examining relationships between central angles, inscribed angles, circumscribed angles and their arcs
- Developing formulas for perimeter and area of regular polygons
- Justifying formulas for circumference and area of circles using intuitive limit arguments
- Practicing circle relationships
- Using proportional reasoning to calculate arc length and area of sectors
- Using the ratio of arc length to radius to develop radians as a way of measuring angles
- Converting between degree measure and radian measure of an angle
- Working with volume and scaling to see relationships

Task: CIRCULAR REASONING

The following problems will draw upon your knowledge of similarity, circle relationships and trigonometry.

In the following diagram the radius of ¤D is 5 cm and F is the midpoint of AE. The measures of arc EB and arc BC are given in the diagram. Find the measures of all other unmarked angles, arcs and segments.

In the diagram below ΔABC is equilateral. All circles are tangent to each other and to the sides of the equilateral triangle. The radius of the three smaller circles, ¤P, ¤Q and ¤R, is 4 cm. The radius of ¤O is not given. Find the circumference and area of each circle and the length of the sides of the equilateral triangle.

-

Students will learn: The purpose of this task is to review and practice theorems and formulas associated with circles. Students will also draw upon ideas of similarity as well as right triangle trigonometry relationships to find the lengths of line segments. 30°V60°V90° triangles appear frequently in this task, so there is an opportunity to emphasize the relationships between the sides and how one can find the exact values of the lengths of the sides in this special right triangle.

Core*Standards*Focus:

G.C.2*(Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Related*Standards:**G.C.3*

Module 8: Circles and Other Conics

Overview: In this module, students will learn to use the Pythagorean Theorem to develop the equation of a coordinate circle. They will use completing the square to find the center & radius of a circle when given a general form equation. They will write the equation of a circle when given a description. Students will write the equation of a parabola when given a description. Finally, students will connect operations on equations of parabolas with operations on circles' equations, and write equations given a description. The tasks covered are:

- Deriving the equation of a circle using the Pythagorean Theorem
- Complete the square to find the center and radius of a circle given by an equation
- Writing the equation of a circle given various information
- Derive the equation of a parabola given a focus and directrix
- Connecting the equations of parabolas to prior work with quadratic functions
- Writing the equation of a parabola with a vertical directrix, and constructing an argument that all parabolas are similar

Task: GETTING CENTERED

Malik's family has decided to put in a new sprinkling system in their yard. Malik has volunteered to lay the system out. Sprinklers are available at the hardware store in the following sizes:

Full circle, maximum 15' radius

Half circle, maximum 15' radius

Quarter circle, maximum 15' radius

All of the sprinklers can be adjusted so that they spray a smaller radius. Malik needs to be sure that the entire yard gets watered, which he knows will require that some of the circular water patterns will overlap. He gets out a piece of graph paper and begins with a scale diagram of the yard. In this diagram, the length of the side of each square represents 5 feet.

1. As he begins to think about locating sprinklers on the lawn, his parents tell him to try to cover the whole lawn with the fewest number of sprinklers possible so that they can save some money. The equation of the first circle that Malik draws to represent the area watered by the sprinkler is:

Draw this circle on the diagram using a compass.

2. Lay out a possible configuration for the sprinkling system that includes the first sprinkler pattern that you drew in #1.

3. Find the equation of each of the full circles that you have drawn.

Malik thought, "That's pretty cool. It's like a different form of the equation. I guess that there could be different forms of the equation of a circle like there are different forms of the equation of a parabola or the equation of a line." He showed his equation to his sister, Sapana, and she thought he was nuts. Sapana, said, "That's a crazy equation. I can't even tell where the center is or the length of the radius anymore." Malik said, "Now it's like a puzzle for you. I'll give you an equation in the new form. I'll bet you can't figure out where the center is." Sapana said, "Of course, I can. I'll just do the same thing you did, but work backwards."

4. Malik gave Sapana this equation of a circle:

$x^2 + y^2 - 4x + 10y + 20 = 0$

Help Sapana find the center and the length of the radius of the circle.

5. Sapana said, "Ok. I made one for you. What's the center and length of the radius for this circle?"

$x2 + y^2 + 6x - 14y - 42 = 06.$

6. Sapana said, "I still don't know why this form of the equation might be useful. When we had different forms for other equations like lines and parabolas, each of the various forms highlighted different features of the relationship." Why might this form of the equation of a circle be useful?

$x2 + y^2 + Ax + By + C = 0$

Students will Learn: The purpose of this task is to solidify understanding of the equation of the circle. The task begins with sketching circles and writing their equations. It proceeds with the idea of squaring the $(x – h)^2$ and $(y – k)^2$ expressions to obtain a new form of an equations. Students are then challenged to reverse the process to find the center of the circle.

Core Standards Focus:

G.GPE Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section

G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Module 9: Probability

Overview: In this module, students will learn to estimate conditional probabilities and interpret the meaning of a set of data. They will examine conditional probability using multiple representations and use samples to estimate probabilities. Students will create Venn diagrams using data while examining data the addition rule for probability. They will examine independence of events using two-way tables and use data in various representations to determine independence. The tasks covered are:

- Estimating conditional probabilities and interpreting the meaning of a set of data
- Examining conditional probability using multiple representations
- Using sample to estimate probabilities
- Creating Venn diagram's using data while examining the addition rule for probability
- Examining independence of events using two-way tables
- Using data in various representations to determine independence

Task: Freddy Revisited

Once Tyrell helped Freddy out in determining the amount and type of food Freddy should prepare each day for his restaurant, Freddy's food waste decreased dramatically. Still, Freddy noticed that during the week, he seemed to still have more food prepared than he needed, and sometimes on the weekend he would run out of something he needed. Tyrell said another level of determining waste could be if Freddy averaged the number of orders he received of fish and chicken on a weekday and compared it to the average number of orders he received of each on the weekend. Freddy thought this was a good idea so started collecting data.

After two months, he had enough information to create a two way table representing the average number of orders he received on the weekdays and on the weekends for fish and chicken.

1. What observations can you make? Explain to Freddy what this means (When does Freddy seem to have the greatest business? Should he expect a greater percentage of customers to order fish during the week or on the weekend? What else?
2. Does the number of orders of chicken compared to fish depend on whether it is a weekday or a weekend? What values from the table tell you this?

Students will Learn: The purpose of this task is for students to determine whether or not an event is independent. In this task, students are asked to interpret the amount of chicken and fish Freddy should prepare on any given day. The goal is for students to recognize that Freddy sales more food on a weekend day than he does on a weekday, however, the percentage of each food type stays the same. In other words, the likelihood that a randomly selected customer would order fish is independent as to whether or not it is a weekday or a weekend.

Core Standards Focus:

S.CP.2: Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S.CP.3: Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S.CP.4: Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. S.CP.5: Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. Related Standards: S.CP.6, S.MD.7

Integrated Math III

Basic Course Information:
Title: Integrated Math III
Length of Course: Full Year
Subject Area: Mathematics (c) / Mathematics III
UC Honors Designation? No
Prerequisite: Integrated Math II
Co-requisites: None
Integrated (Academics / CTE): No
Grade Levels: 10th, 11th & 12th
Course Description: Overview: Integrated Math 3 is the third course of a three-course sequence including Integrated Math 1, 2, and 3. This course is aligned with the Common Core standards for Integrated Math 3. The content standards for Integrated Math 3 and standards for mathematical practice can be viewed on the CDE website. Content standards for Integrated Math 3 include: 1) Extending understanding of all functions including inverse functions, logarithmic functions, polynomial functions of degree higher than 2, rational functions and trigonometric functions. 2) Develop understanding of log properties and use them to evaluate various logarithmic expressions. 3) Modeling by use of geometry with all types of triangles and various solids. 3) Transformation of all types of functions from multiple perspectives. 4) Understanding and use of Trigonometric Functions using a variety of modalities. 5) Develop understanding of statistical functions beyond linear models. In this course, students review and develop skills learned Integrated Math 1 and 2 and proceed into higher level mathematical reasoning, teaching them to understand and apply mathematical concepts and tools in the following ways; graphically, numerically, algebraically, and in written and

spoken presentations. This course will also show physical and realistic application of mathematics and how mathematics is a mechanism for problem solving in many areas of life (from personal finances to workplace applications.) Students who are successful in this course will be advanced to either pre-Calculus or Statistics.

There are 8 modules to this class. **Each module consists of the following:**

- **Classwork Task**: Launch – whole class, Explore – individual, pairs or small groups, Discuss – whole class
- **Homework**: assignments have been designed to continue to spiral a review of content.
- **Quizzes**: two quizzes per module consisting of multiple-choice, free response, and extended response items.
- **End of Module Exam**: one exam per module consisting of multiple-choice, free response, and extended response items.

Module 1: Functions and Their Inverses

Develops the concept of inverse functions in a linear modeling context using tables, graphs, and equations.

Extends the concepts of inverse functions in a quadratic modeling context with a focus on domain and range and whether a function is invertible in a given domain.

Solidifies the concepts of inverse function in an exponential modeling context and surfaces ideas about logarithms.

Uses function machines to model functions and their inverses. Focus on finding inverse functions and verifying that two functions are inverses.

Uses tables, graphs, equations, and written descriptions of functions to match functions and their inverses together and to verify the inverse relationship between two functions.

1.3 Tracking the Tortoise - A Solidify Understanding Task

Purpose: The purpose of this task is two-fold:

1. To extend the ideas of inverse to exponential functions
2. To develop informal ideas about logarithms based upon understanding inverses. (These ideas will be extended and formalized in Module II: Logarithmic Functions)

The first part of the task builds on earlier work in Secondary I and II with exponential functions.

Students are given an exponential context, the distance travelled for a given time, and then asked to reverse their thinking to consider the time travelling for a given distance. After creating a graph of the situation, students are asked to consider the extended domain of all real numbers and think about how that affects the inverse function. Questions at the end of the task press on understanding of the inverse relationship and the idea that a function and its inverse “undo” each other, using function notation and particular values of a function and its inverse.

Core Standards Focus:

F.BF.1 Write a function that describes a relationship between to quantities.

F.BF.4 Find inverse functions.

1. Read values of an inverse function from a graph or a table, given that the function has an inverse.

F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Launch (Whole Class): Begin the task by being sure that students understand the problem situation. (If students have done Module I in Secondary Mathematics II, they will be familiar with the context.) Ask students to work individually on problems 1 and 2. Briefly discuss the answers to question 1 and be sure that students know how to use the model given. Then, ask students to describe what they know about d(t) before they graph the function. Be sure to include features such as: continuous, y-intercept (0,1), always increasing, and the end behavior, as _

Use technology to draw the graph and confirm their predictions about the graph.

Explore (Small Group): Have students work on the rest of the task. Questions 3 and 4 help to draw them into the inverse context, much like the work in the previous two tasks. If students are initially stuck, ask them what strategies they used to draw the graph in Flipping Ferraris.

Encourage the use of tables to generate points on the graph. Students should use their graph to estimate a value for question #6—they do not yet have experience with logarithms to solve an equation of this type. Questions 8-11 become more abstract and rely on their previous work with inverses. You may wish to start the discussion when most students have finished question #7 and discuss problems 3-7, before directing students to work on 8-11 and then discussing those problems.

Discuss (Whole Class): Begin the discussion by asking a student to present a table of values for the graph of t(d). Ask student to describe how they selected the values of d to elicit the idea that they chose powers of 2 because they were easy to think about the time. This strategy will become increasingly important in Module II, so it is important to make it explicit here. Draw the graph of the inverse function t(d) and ask students how it compares to the graph of d(t). They should again notice the reflection over y = x. Ask students to compare the domain and range of d(t) with the domain and range of t(d). As students to discuss the graphs and conclude that t(d) is the inverse function of d(t), ask them to begin to generalize the patterns they see about inverses to all functions.

Some questions you could ask are:

Will a function and its inverse always be reflections over the y = x line? Why?

Will the domain of the function always be the range of the inverse function? Why or why not?

Student Part: You may remember a task from last year about the famous race between the tortoise and the hare. In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race. In the task, we modeled the distance from the starting line that both the tortoise and the hare travelled during the race. Today we will consider only the journey of the tortoise in the race. Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise including the head start is given by the function: see function (d in meters and t in seconds) The tortoise family decides to watch the race from the sidelines so that they can see their darling tortoise sister, Shellie, prove the value of persistence. 1. How far away from the starting line must the family be, to be located in the right place for Shellie to run by 10 seconds after the beginning of the race? After 20 seconds? 2. Describe the graph of d(t), Shellie's distance at time t. What are the important features of d(t)? 3. If the tortoise family plans to watch the race at 64 meters away from Shellie's starting point, how long will they have to wait to see Shellie run past? 4. How long must they wait to see Shellie run by if they stand 1024 meters away from her starting point? 5. Draw a graph that shows how long the tortoise family will wait to see Shellie run by at a given location from her starting point. 6. How long must the family wait to see Shellie run by if they stand 220 meters away from her starting point? 7. What is the relationship between d(t) and the graph that you have just drawn? How did you use d(t) to draw the graph in #5? 8. Consider the function 9. A) What are the domain and range? Is function invertible? 10. B) Graph the function and the inverse on the grid below. 11. C) What are the domain and range of inverse function
Module 2: Logarithmic Functions
Evaluate and compare logarithmic expressions. Graph logarithmic functions with transformations Develops understanding of log properties Use log properties to evaluate expressions **2.2 Falling Off A Log – A Solidify Understanding Task**

Note to Teachers: Access to graphing technology is necessary for this task.

Purpose: The purpose of this task is to build on students' understanding of a logarithmic function as the inverse of an exponential function and their previous work in determining values for logarithmic expressions to find the graphs of logarithmic functions of various bases. Students use technology to explore transformations with log graphs in base 10 and then generalize the transformations to other bases.

Core Standards Focus:

F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Launch (Whole Class): Begin class by reminding students of the work they did with log expressions in the previous task and soliciting a few exponential and log statements like this:

log5 125 = 3

Encourage the use of different bases to remind students that the same definition works for all bases, b > 1. Tell students that in this task they will use what they know about inverses to help them create tables and graph log functions.

Explore (Small Group): Monitor students as they work to see that they are completing both tables and graphs for each function. Some students may choose to graph to exponential function and then reflect it over the y = x line to get the graph before completing the table. Watch for this strategy and be prepared to highlight it during the discussion. Finding points on the graph for 0 <_ < 1 may prove difficult for students since negative exponents are often difficult. Remind them that it may be easier to find points on the exponential function and then switch them for the log graphs if they are stuck.

Discuss (Whole Class): Begin the discussion with the graph of _ (_) = log2_. Ask a student that used the exponential function _ = 2 and switched the x and y values to present their graph. Then have a student that started by creating a table describe how they obtained the values in the table.

Ask the class to identify how the two strategies are connected. By now, students should be able to articulate the idea that powers of 2 are easy values to think about and that the value of the log expression will be the exponent in each case.

Move the discussion to question #4, the similarities between the graphs. Students will probably speak generally about the shapes being alike. In the discussions of similarities, be sure that the more technical features of the graphs emerge:

Student's questions:

1. Construct a table of values and a graph for each of the following functions. Be sure to select at least two values in the interval 0 < x < 1.
2. How did you decide what values to use for x in your table?
3. How did you use the x values to find the y values in the table?
4. What similarities do you see in the graphs?
5. What differences do you observe in the graphs?
6. What is the effect of changing the base on the graph of a logarithmic function?

Test your prediction by graphing log for various values of b. 1. What is the effect of adding b? 2. What will be the effect of subtracting b? 3. Make a general argument for why this is true for all logarithmic functions. 7. Write an equation for each of the following functions that are transformations of (see graph). 8. Compare the transformation of the graphs of logarithmic functions with the transformation of the graphs of quadratic functions. Note: There are more questions and information given to students along with graphs, but these will not copy into the allowable text for the submission. Please see Module 2 student work on the website for more details.
Module 3: Polynomial Functions
Comparing growth rates of linear, quadratic, and cubic functions and recognizing that cubic functions can be created from the sums of a quadratic function Determining the slowest to the fastest growing functions by ordering and comparing values as x Approaches infinity Understanding end behavior and comparing end behavior of functions in different representations Using graphical representations to add, subtract, and multiply polynomials Determining the nature of roots and applying the Fundamental Theorem of Algebra. Part II Experiments with the Binomial Expansion using Pascal's Triangle Applying the Fundamental Theorem of Algebra Using the Remainder Theorem to find all linear factors and roots of a polynomial function Practicing all things related to graphing and solving polynomial functions **3.4 Polynomial Connections! – A "Solidify" Understanding Task**

Purpose: The purpose of this task is to have students make connections of polynomial functions.

The first section encourages students to see the results of adding and subtracting polynomial functions. As students progress through the task, they will become more comfortable using graphs to add, subtract, and multiply polynomials graphically. After completing the graphs, students should be able to answer question 10 with statements similar to the following:

"A linear plus a linear is a linear"

"A linear plus a quadratic is a quadratic"

"The product of two linear functions is a quadratic"

"The x B intercepts remain the same when multiplying two functions"

Aside from these statements, also be sure to talk about end behavior. We want students to make connections to how polynomials behave similar to integers in general. (For example, when multiplying two positive sloped linear functions, the result is a positive quadratic function.)

Note to teacher: If students are not used to combining functions graphically, have them work on this task in partners. You may even wish to already have the original functions graphed so that they are focusing on the operation and the resultant graph.

Standards Focus:

A.APR.1 Understand that polynomials for a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

F.BF.1: Write a function that describes a relationship between two quantities.

1. Combine standard function types using arithmetic operations.

Related standards: F.IF.4

Launch (Whole Class): Start this task by explaining to students that they are going to use graphs to add, subtract, and multiply polynomial functions. As they complete each problem, have them write down observations. If students are not used to combining functions graphically, have them work on the task in partners.

Explore (Small Group/Pairs): Monitor student thinking as students work. If students are stuck, ask them to determine the output value of f+g!at x!= 0 (while you point to the graphs at x = 0). After students compute values at a few locations, they will begin to see the pattern. If the whole class seems stuck, you may wish to create an example to discuss with the whole class (such as graphing y = x+3 and y = 2). Do not go through all of the observations, just make sure they have a process to find solutions.

As students write observations about the first problem, press for them to notice more than one thing. For example, question 1 observations could include that the resulting graph is linear, has a steeper slope than the original graphs (getting at the connection that a positive plus a positive is 'more positive'), that the intercepts of the original graph is also where the resultant graph intersects the other (a+0=a), etc. Students do not have to make all of these observations, but the more they make for question 1, the better they will get as they progress through the problems (plus it will add to the whole group discussion). As you begin to select students to share out, pay

attention to HOW students are creating the resultant graph as well as to the observations being made. **Discuss (Whole Class):** The whole group discussion should lead to students seeing that polynomials are analogous to integers. The last question of the task should be easy to answer if students are making observations throughout the task. Begin the whole group discussion after you feel all students have made at least one observation and that between the groups of students that all of the statements below can be made: “A linear plus a linear is a linear” “A linear plus a quadratic is a quadratic” “The product of two linear functions is a quadratic” “The x B intercepts remain the same when multiplying two functions” Select students to share based on observations and their justification of the observation. Prior to sharing with the whole class, prompt them that they are going to write their observation about combining functions and then use the work they have to justify and explain their observation. For example, if a student says that the sum of two linear functions is a linear function… then have them write this statement and then use their work to explain to the class why this is so. Focus on these observations: When adding/subtracting: “A linear plus a linear is a linear” and/or “A linear plus a quadratic is a quadratic” and/or “The end behavior is like when you are adding or subtracting integers” When multiplying: “The product of two linear functions is a quadratic” and/or “The product of three linear functions is a cubic” and/or “The product of a quadratic and a linear function is a cubic” and/or “The x B intercepts remain the same when multiplying two functions” and/or “The end behavior is like when you are multiplying integers”
Module 4: Rational Functions
Using context to identify inverse variation and introduce rational functions Analyzing the characteristics of various families of functions to assist in identifying characteristics of Rational functions. Connecting rational expressions to rational numbers Connecting rational numbers: improper fractions to rewrite improper rational Identifying the end behavior of rational functions

Graphing rational functions using features of rational functions

Graphing and solving rational functions

4.5 Watch Your "Behavior"

In this task, you will develop your understanding of the end behavior of rational functions as well as discover the behavior of even and odd functions.

Part I: End Behavior of Rational Functions

After completing the task, The Gift, Marcus and Hannah were talking about the discussion regarding the end behavior of the parent function f(x) = 1/x, Marcus said "I thought the end behavior of all functions was that you either ended up going to positive or negative infinity." Hannah agreed, adding " Now we have a function that approaches zero. I wonder if all rational functions will always approach zero as x approaches + or – infinity." Marcus replied "I am sure they do. Just like all polynomial functions end behavior approaches + or – infinity, I think the end behavior for all rational functions must approach zero".

1. Could Marcus be right? Make a conjecture about the end behavior of rational functions and test it. (Hint: this should take awhile-be sure to think about the various rational expressions we have studeied). As you analyze the end behavior of different rational functions, try to generalize the patterns you notice regarding end behavior.

Purpose: The purpose of this task is for students to surface their thinking about the end behavior Asymptote of rational functions (both proper and improper) and also to examine whether a Function is even, odd, or neither.

Teacher note: Prior to this task, students have become familiar with the end behavior of various Functions and have also become familiar with proper and improper rational expressions. In this task, we are asking students to think about what conditions make it so that the end behavior Asymptote is *y =0,* and are there times when it would be something else? Can they explain under What conditions the end behavior asymptote would be something different? It is important to note Here that while you want to press your students to come up with different examples and to try to

Find examples of rational functions with different end behaviors, if your class does not generate all

Three scenarios for end behavior in this task, it is ok as the next few tasks solidify this idea.

In Part II, students are introduced to even and odd functions and will generate definitions for even

And odd functions based on observations provided (using multiple representations).

Standards Focus:

F.IF.7d Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

1. Graph rational functions, identifying zeros when suitable factorizations are available, and showing end behavior.

F.BF.3 Identify the effect on the graph of replacing *f(x) by f(x+k, kf(x), f(kx), and f(x+k)* for specific

Values of k (both positive and negative); find the value of *k* given the graphs. Experiment with cases

And illustrate an explanation of the effects on the graph using technology. Include recognizing even

And odd functions from the graphs and algebraic expressions for them.

Launch Part I (Whole Class): Start this task by reading the scenario and then letting students

Know that they should work in partners and try to come up with various rational functions that

Create different end behaviors. For each rational function they create, they should state the end behavior and come up with an equation that models the end behavior asymptote. They should also

Be able to provide evidence that their end behavior is correct.

Explore Part I(small groups): Monitor student thinking as they work in small groups. Press

Students to first create different rational functions and explain the end behavior, then see if they

Can identify why the end behavior for a particular function exists (moving them to a generalization

To find the end behavior asymptotes for various rational functions). For students who struggle, ask

Them to start simple and create a rational function, explain why it is rational and then find/explain

The end behavior. Tell them to repeat the process by changing the function a bit and see if this Changes the end behavior.

As you monitor, look for students who notice the end behavior goes to either positive or negative Infinity when the degree of the numerator is higher than the denominator, for students who notice The end behavior is a constant when the degree of both the numerator and denominator are the Same (and that the constant is the coefficients), for students who can explain that the end behavior Is always going to zero when the degree of the numerator is less than the denominator, and also Look for students who may extend this thinking to connect to the work they did in the task 4.4 Rational Expressions and come up with the end behavior asymptote equation. In each situation, Make sure the students can explain/provide evidence of their claim. It is more important in this Task that they know what different rational functions look like and that they can determine the end behavior than it is that they solidify their understanding of end behavior asymptotes of all rational functions.

Discuss Part I(Whole Class): Once students have done their best, bring the whole group Together to share their findings. One way to sequence student work may be to select a couple of Students to share that have come up with different equations and yet have the same end behavior (those with an end behavior of positive infinity or negative infinity). Does everyone in the class Agree that the end behavior is the same for these equations? How are these equations similar? At This time, focus on the end behavior and not the asymptote equation. Next, select a student whose Function end behavior is a constant and have them explain why. The extent of this whole group Discussion depends on what ideas and findings your students came up with.

Part II: Even and Odd Functions

Below are 3 graphs: one represents an even function, one represents an odd function, and one is neither even nor odd.

2. Use the graphs and their corresponding functions to write a definition for an even function and an odd function.

A function is even if…

A function is odd if…

Launch Part II (Whole Class): After the whole group discussion from Part I, have students Look at the three functions in Part II and have them write down their ideas for what they think the Definition is for an even function, an odd function, and a function that is neither even nor odd. Gather ideas from the group, then have them work in their small groups to answer question 3.

Explore Part II (Small Group): Monitor students as they work. When appropriate, let them Know the answers to question 3 are at the bottom of the page so they can check to see if their Solutions match. Redirect students to adjust their definition for even function, odd function, and Those that would be neither based on what they have learned.

Discuss (Whole Class): For the whole group discussion, have the class clarify the definitions. Make sure the definition includes the following:

EVEN:

- A function *f(x)* is classified as an **even function** if it is symmetric about the y axis.
- A function *f(x)* is classified as an **even function** if the output is the same for both *x and –x.*

(in other words, a function is an even function if $f(-\theta) = f(\theta)$).

ODD:

- A function *f(x)* is classified as an **odd function** if it is symmetric about the origin (or rotates

180 degrees about the origin onto itself or reflects about the *x and y axis* onto itself).

- If a **function** is an odd function, then is the point (*a, b)* satisfies the function, then so does

The point (*-a, -b).*

- A function *f(x)* is classified as an **odd function** if $f(-\theta) = -f(\theta)$.

IF a function is not even or odd, then it is neither.

Module 5: Modeling with Geometry

Visualizing two dimensional cross sections of three dimensional objects

Visualizing solids of revolution

Approximating volumes of solids of revolution with cylinders and frustums

Solving problems using geometric modeling

Examining the relationship of sides in special right triangles

Developing strategies for solving non right triangles

Examining the Law of Cosines and the Law of Sines

Finding the missing sides, angles and area of general triangles

5.5 Special Rights –A"Solidify"Understanding"Task"

Purpose: This task allows students the opportunity to review previous work with right triangles using the Pythagorean theorem and right triangle trigonometry in preparation for the next task in this module where they will find ways to determine unknown measurements in nonNright triangles.

In this task students will develop relationships between the lengths of the sides of 45°45°90° triangles and 30°60°90° triangles, in preparation for finding exact values for the coordinates of points on the unit circle in module 6.

Core Standards Focus:

G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non right triangles (e.g., surveying problems, resultant forces).

Launch (Whole Class):

Give students a few minutes to individually answer questions 1 and 2, and then discuss them as a whole class. Students should be able to identify that to use the Pythagorean theorem we need to know any two sides of the right triangle, and to use right triangle trig we need to know one side and the measure of an acute angle. We select the appropriate trig ratio depending on the given side and the side to be determined.

Once students have reviewed these key issues, read with them the paragraph that describes special "right" triangles, then set students to work on the task."

Explore (Small Group):

Questions 3 and 5 ask students to visualize special right triangles as part of another, well known geometric shape. Do not shortchange this visualization by telling students how to “see” these triangles or what to relate them to. Students should be allowed to develop these relationships for themselves, so they will recognize them in the future. For many students it is difficult to recall these relationships, but it is easy to re:derive "them when needed. If needed, prompt students by asking, “What if you put two congruent copies of these triangles together, what shapes might be formed? What can you say about the side lengths in this new figure? What happens to the side lengths when the two copies are pulled apart?”

Listen for students who can present the more general strategies for relating a side of length x"in a special right triangle to the lengths of the other sides. You may prompt this discussion by asking, “If the length of this side is x, how might we express the length of the other two sides in terms of x?”

Discuss (Whole Class):

Select a student to present who can relate a 45°N45°N90° triangle to the triangle formed by a diagonal of a square, and then use that image to reason about the lengths of the other two sides. All students should make note of both of the following possible scenarios, and be able to derive the other sides based on the side whose length is x."

Select a student to present who can relate a 30°N60°N90° triangle to the triangle formed by an altitude of an equilateral triangle, and then use that image to reason about the lengths of the other two sides. All students should make note of each of the following possible scenarios, and be able to derive the other sides based on the side whose length is x."

Student work:

In previous courses you have studied the Pythagorean theorem and right triangle trigonometry. Both of these mathematical tools are useful when trying to find missing sides of a right triangle.

1. What do you need to know about a right triangle in order to use the Pythagorean theorem?
2. What do you need to know about a right triangle in order to use right triangle trigonometry?

While using the Pythagorean theorem is fairly straight forward (you only have to keep track of the legs and hypotenuse of the triangle), right triangle trigonometry generally requires a calculator to look up values of different trig ratios. There are some right triangles, however, for which knowing a side length and an angle is enough to calculate the value of the other sides without using trigonometry. These are known as special "right" triangles because their side lengths can be found by relating them to another geometric figure for which we know a great deal about its sides.

One type of special right triangle is a 45°45°90° triangle.

3. Draw a 45°45°90° triangle and assign a specific value to one of its sides. (For example, let one of the legs measure 5 cm, or choose to let the hypotenuse measure 8 inches. You

will want to try both approaches to perfect your strategy.) Now that you have assigned a measurement to one of the sides of your triangle, find a way to calculate the measures of the other two sides. As part of your strategy, you may want to relate this triangle to another geometric figure that may be easier to think about.

4. Generalize your strategy by letting one side of the triangle measure x. Show how the measure of the other two sides can be represented in terms of x.""(Make sure to consider cases where x is the length of a leg, as well as the case where x is the length of the hypotenuse.)

Another type of special right triangle is a 30°60°90° triangle.

5. Draw a 30°60°90° triangle and assign a specific value to one of its sides. Now that you have assigned a measurement to one of the sides of your triangle, find a way to calculate the measures of the other two sides. As part of your strategy, you may want to relate this triangle to another geometric figure that may be easier to think about.
6. Generalize your strategy by letting one side of the triangle measure x. Show how the measure of the other two sides can be represented in terms of x. (Make sure to consider cases where x is the length of a leg, as well as the case where x is the length of the hypotenuse.)

7. Can you think of any other angle measurements that will create a special right triangle?

Module 6: Trigonometric Functions

Using reference triangles, right triangle trigonometry and the symmetry of a circle to find the y-coordinates of points on a circular path

Using reference triangles, right triangle trigonometry, angular speed and the symmetry of a circle to find the y-coordinates of points on a circular path at given instances in time—an introduction to circular trigonometric functions

Extending the definition of sine from a right triangle trigonometry ratio to a function of an angle of rotation

Graphing a sine function to model circular motion and relating features of the graph to the

Parameters of the function

Extending the definition of cosine from a right triangle trigonometry ratio to a function of an angle of rotation

Introducing radians a unit for measuring angles on concentric circles

Using the proportionality relationship of radian measure to locate points on concentric circles

Redefining radian measure of an angle as the length of the intercepted arc on a unit circle

Defining sine and cosine on the unit circle in terms of angles of rotation measured in radians

Introducing the horizontal shift of a trigonometric function in terms of a modeling context

Using trigonometric graphs and inverse trigonometric functions to model periodic behavior

Practice using transformations of trigonometric graphs and inverse trigonometric functions to model periodic behavior

Extending the definition of tangent from a right triangle trigonometry ratio to a function of an angle of rotation, including angles of rotation measured in radians on the unit circle; classifying sine, cosine and tangent functions as even or odd

6.1 George W. Ferris' Day Off – A"Develop"Understanding"Task"

Purpose: The purpose of this task is to help students visualize a way that right triangle trigonometric ratios can be used as a tool to describe periodic behavior—in this case, rotation around a Ferris wheel. In the next task students will consider the motion of a point around a moving Ferris wheel. In this task students consider points on a stationary Ferris wheel and determine how they can find the height of those points above the ground.

Imagining the spokes of the Ferris wheel as the hypotenuse of various right triangles—triangles drawn by dropping a vertical line from the endpoints of the spokes to the horizontal line through the center of the Ferris wheel—will allow students to use the sine ratio to find the distance of a point on the wheel above or below the center of the wheel. Students will develop a strategy for finding the height of any point: height"="30"+"25 sin(θ), where θ is the angle formed by the horizontal line through the center of the wheel to a particular spoke of the wheel. Since students are only familiar with right triangle trigonometry, θ is between 0° and 90°. Consequently, students will need to consider how to find θ when the endpoint of the spoke of interest lies in quadrants II, III or IV.

Students will also observe the symmetry of points around the wheel as a way to reduce the number of computations needed to find the heights of all ten endpoints. This task develops essential ways of thinking about the location of points around a circle that will become fundamental in students' understanding of trigonometric functions, radian measure and the unit circle.

Core Standards Focus:

F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Launch (WholeClass):

After introducing the scenario from the introductory paragraphs of this task, ask students to individually answer questions 1, 2 and 3. After students have had a few minutes to think about the structure of the Ferris wheel, have a couple of students discuss how they could answer these questions. It is important to surface the idea that distances from the ground to any point on the

Ferris wheel can be found by adding to or subtracting from the height of the center of the wheel.

Remind students of what they know about right triangle trigonometry from Secondary Math I, particularly the definitions of the trigonometric ratios: for an acute angle of a right triangle, the sine of that angle is the ratio of the length of the side opposite the angle to the length of the hypotenuse; the cosine of that angles is the ratio of the length of the side adjacent to the length of the hypotenuse; and, the tangent of the angle is the ratio of the length of the side opposite to the length of the side adjacent. Once you have reviewed these definitions, set students to work on finding the distance from the ground to each of the endpoints of the ten spokes of the wheel.

Explore (Small Group):

Students may struggle with how to get started, since there are no right triangles drawn in the diagram. Let students struggle with this for a while, perhaps asking such questions as, "Where might you draw a right triangle on this diagram so you would know something about its sides or angles?" or "What do you know for sure about the triangle you just drew?"

Students may make several attempts at drawing right triangles until they find one for which they do know a length and an angle. Since it can be determined that the angle between two spokes measures 36° (one tenth of the circle), and the length of a spoke is 25 feet (the radius of the wheel), there are two potential triangles students might draw for which they do know enough information to apply right triangle trigonometry, as shown in this diagram. (Diagram cannot be copied here but is available on mathematics vision project website lesson 6.5 teacher notes.)

Students are often so pleased to find a triangle they can apply trigonometry to, that they do not recognize that the triangle that uses a spoke as a leg of the right triangle does not give them information about the height of an endpoint above or below the center of the wheel. Allow them to struggle with this idea, perhaps by asking the question, "And what would this triangle look like if you were to draw it for another endpoint?"

Since students know they want to find the distance from the ground to a point, watch for students that start by drawing a line segment from an endpoint of a spoke perpendicular to the ground. Listen for students who suggest that they might decompose this line segment into two parts: the part which measures 25 feet from the ground to the line that passes horizontally through the center, and the part that lies above the horizontal line whose length will need to be calculated. Once students are focused on this segment that lies above the horizontal line through the center, suggest they try to draw a right triangle that includes this line segment. Watch to see if they draw the right triangle to include this line segment and the center of the wheel as one of its vertices. If so, this should lead them to a strategy that involves using the sine ratio since they will be calculating the length of a side opposite of the angle whose vertex is at the center of the Ferris wheel.

Some students may draw a triangle, such as shown in the diagram, for which they can use the cosine ratio to find the distance above or below the center of the wheel. While this strategy works, it seems less straight forward than the strategy described in the previous paragraph, since we are determining the height of a point above the ground by comparing it to another point with the same height, and finding the measure of an acute angle in the triangle is not as straight forward as just noting the angle between the spokes.

Some students will change strategies from quadrant to quadrant, sometimes using a sine ratio in one quadrant and a cosine ratio in another. During the whole class discussion you will want to discuss the value of finding a single strategy that works in all cases, and help students recognize that using the sine ratio seems straighter forward in terms of the location of the point relative to the ground.

Discuss (Whole Class):

Have students share strategies for finding the height of each endpoint of the spokes on the Ferris wheel. Start by labeling the height of the points farthest to the right and left of the wheel—points A and F—since they line along a horizontal line through the center of the wheel. Next, have someone present how he found the height at point B, and then have someone present her work for point C.

Select students to present that used a sine ratio to calculate the height of each of these points.

As you move around the circle, select students to present who calculated the heights of points D, E, G, H, I and J, as well as students who used the symmetry of the circle to determine the heights.

Lead students to generalize that the height of any endpoint of a spoke can be found using the formula height"="30"+"25sin(θ), where θ is the angle formed by the horizontal line through the center of the wheel and the line segment that represent a particular spoke of the wheel.

If some students changed strategies from point to point or quadrant to quadrant, discuss the value of finding a single strategy that works in all cases. If some students used a cosine ratio point out that it is a correct strategy, but suggest that using the sine ratio seems more straight forward in

terms of the location of the point relative to the ground, since the endpoints of the spokes do not lie on the vertical line through the center of the wheel. **Student work:** Perhaps you have enjoyed riding on a Ferris wheel at an amusement park. The Ferris wheel was invented by George Washington Ferris for the 1893 Chicago World's Fair. Carlos, Clarita and their friends are celebrating the end of the school year at a local amusement park. Carlos has always been afraid of heights, and now his friends have talked him into taking a ride on the amusement park Ferris wheel. As Carlos waits nervously in line he has been able to gather some information about the wheel. By asking the ride operator, he found out that this wheel has a radius of 25 feet, and its center is 30 feet above the ground. With this information, Carlos is trying to figure out how high he will be at different positions on the wheel. 1. How high will Carlos be when he is at the top of the wheel? (To make things easier, think of his location as simply a point on the circumference of the wheel's circular path.) 2. How high will he be when he is at the bottom of the wheel? 3. How high will he be when he is at the positions farthest to the left or the right on the wheel? Because the wheel has ten spokes, Carlos wonders if he can determine the height of the positions at the ends of each of the spokes as shown in the diagram. Carlos has just finished studying right triangle trigonometry, and wonders if that knowledge can help him. 4. Find the height of each of the points labeled ASJ on the Ferris wheel diagram on the following page. Represent your work on the diagram so it is apparent to others how you have calculated the height at each point. Final NOTE: Many key assignments contain special graphics including equations, symbols and graphs that will not download to this text box. Please go to www.mathematicsvisionproject.org to view the Integrated 3 Curriculum these lessons came from to view the graphics and other key assignments that could not be placed in this text box.
Module 7: Modeling with Functions
Examining the transformations of a variety of familiar functions using tables Predicting the shape of a graph that is the sum or product of familiar functions Combining a variety of functions using arithmetic operations to model complex behavior

Combining a variety of functions using function composition to model complex behavior

Examining function transformations by composing and decomposing functions

Combining functions defined by tables, graphs or equations using function composition and/or arithmetic operations

7.3 The Bungee Jump Simulator

A Solidify Understanding Task

As a reward for helping the engineers at the local amusement park select a design for their next ride, you and your friends get to visit the amusement park for free with one of the engineers as a tour guide. This time you remember to bring your calculator along, in case the engineers start to speak in “math equations” again.

Sure enough, just as you are about to get in line for the Bungee Jump Simulator, your guide pulls out a graph and begins to explain the mathematics of the ride. To prevent injury, the ride has been designed so that a bungee jumper follows the path given in this graph. Jumpers are launched from the top of the tower at the left, and dismount in the center of the tower at the right after their up and down motion has stopped. The cable to which their bungee cord is attached moves the rider safely away from the left tower and allows for an easy exit at the right.

Your tour guide won’t let you and your friends get in line for the ride until you have reproduced this graph on your calculator exactly as it appears in this diagram.

1. Work with a partner to try and recreate this graph on your calculator screen. Make sure you pay attention to the height of the jumper at each oscillation, as given in the table.

Record your equation of this graph here:

After a thrilling ride on the Bungee Jump Simulator, you are met by your host who has a new puzzle for you. “As you are aware,” says the engineer, “temperatures around here are very cold at night, but very warm during the day. When designing rides we have to take into account how the metal frames and cables might heat up throughout the day. Our calculations are based on Newton’s Law of Heating. Newton found that while the temperature of a cold object increases when the air is warmer than the object, the rate of change of the temperature slows down as the temperature of the object gets closer to the temperature of its surrounding.”

Of course the engineer has a graph of this situation, which he says "represents the decay of the difference between the temperature of the cables and the surrounding air."

Your friends think this graph reminds them of the points at the bottom of each of the oscillations of the bungee jump graph.

2. Using the clue given by the engineer, "This graph represents the decay of the difference between the temperature of the cables and the surrounding air," try to recreate this graph on your calculator screen. (Hint: What types of graphs do you generally think of when you are trying to model a growth or decay situation? What transformations might make such a graph look like this one?)

Record your equation of this graph here:

Purpose:

The purpose of this task is to solidify thinking about combining function types using such operations as addition or multiplication. In the previous task students have noticed how the characteristics of both types of functions are manifested in the resulting graphs. In this task they become more precise about how the functions combine by trying to determine the exact equation that will produce a given graph.

Core Standards Focus:

F.BF.1b

Write a function that describes a relationship between two quantities.

Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Module 8: Statistics

Understand normal distributions and identify their features.

Use the features of a normal distribution to make decisions.

Compare normal distributions using z scores.

Compare normal distributions using z scores and understanding of mean and standard deviation.

Understand and identify different methods of sampling.

Uses tables, graphs, equations, and written descriptions of functions to match functions and their inverses together and to verify the inverse relationship between two functions.

Identify the difference between survey, observational studies, and experiments.

Use simulation to estimate the likelihood of an event.

8.2 Just ACT Normal

A Solidify Understanding Task

1. One of the most common examples of a normal distribution is the distribution of scores on standardized tests like the ACT. In 2010, the mean score was 21 and the standard deviation was 5.2

(Source: National Center for Education Statistics). Use this information to sketch a normal distribution curve for this test.

2. Use technology to check your graph. Did you get the points of inflection in the right places?

(Make adjustments, if necessary.)

3. In "What Is Normal", you learned that the 68 – 95 – 99.7 rule. Use the rule to answer the following questions:
4. What percentage of students scored below 21?
5. About what percentage of students scored below 16?
6. About what percentage of students scored between 11 and 26?
7. Your friend, Calvin, would like to go to a very selective college that only admits the top 1% of all student applicants. Calvin has good grades and scored 33 on the test. Do you think that Calvin's ACT score gives him a good chance of being admitted? Explain your answer.

5. Many students like to eat microwave popcorn as they study for the ACT. Microwave popcorn producers assume that the time it takes for a kernel to pop is distributed normally with a mean of 120 seconds and a standard deviation of 13 for a standard microwave oven. If you're a devoted popcorn studier, you don't want a lot of un-popped kernels, but

you know that if you leave the bag in long enough to be sure that all the kernels are popped, some of the popcorn will burn. How much time would you recommend for microwaving the popcorn? Use a normal distribution curve and the features of a normal distribution to explain your answer.

Purpose: The purpose of this task is to recognize a normal distribution. In the task, students are

Given multiple examples of frequency distributions, some normal and some that are not. Students

Are asked to compare the examples and identify features of a normal distribution. The purpose is to understand that the shape of a normal distribution is symmetric, single P peaked, and bell shaped. Students will recognize the effect on the distribution curve of changing the mean and the standard deviation. They will also learn that the mean, median, and mode are equal in a normal distribution. Students will also see the 68 – 95 - 99.7 rule illustrated, and then will use the rule in a later task.

Core Standards Focus:

S.ID.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to

Estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal

curve.

Related Standards:S.ID.1

Pre-Calculus Honors

Basic Course Information:
Title: Pre-Calculus Honors
Length of Course: Full Year
Subject Area: Mathematics (c) / Advanced Mathematics
UC Honors Designation? Yes
Prerequisite: Integrated Math III
Co-requisites: None
Integrated (Academics / CTE): No
Grade Levels: 11th & 12th
Course Description: Honors Pre Calculus is accelerated, covering all topics in the regular Pre Calculus course, then advancing through a basic introduction of the concepts of Limit, Instantaneous Rate of Change, and Definite Integral. Students will acquire the ability to complete work on the practical application of these ideas. In addition, students are provided more thorough practice with elementary sequences, series, and summation notation. Students will be able to utilize advanced technologies to assist in solving problems, as well as will be able to apply material learned in class to real life examples. The purpose of the course is to advance student

knowledge of mathematics, in order for them to become excellent problem solvers allowing them to explore ideas in depth and develop strong problem-solving skills in a variety of applications.
Functions and Their Graphs
Students will be able to: • Evaluate functions and find their domains. • Analyze graphs of functions. • Identify and graph shifts, reflections, and non-rigid transformations of functions. • Find arithmetic combinations and compositions of functions. • Find inverses of functions graphically and algebraically. • Verify by composition that one function is the inverse of another • Read values of an inverse function from a graph or a table, given that the function has an inverse. • Produce an invertible function from a non-invertible function by restricting the domain. The unit begins with a review of functions and how they can be used to model change, as well as a brief review of the characteristics of linear, exponential, and quadratic functions from Algebra I and II. The topic then extends students' understanding of the principles of linear functions to explore how rates of change can be used to choose an appropriate function model. Students will be expected to model scenarios using an appropriate function model, and to justify their selection in writing. Next, students will look at combinations of different types of families of functions. These combinations include sums, differences, products, quotients and piecewise defined functions. Students will start to refine their understanding of domain during this part of the unit. Students model situations from physics and economics throughout the topic. After studying combinations, students will study composition of functions. They apply compositions to problem situations, including multiple discounts on sale items and natural disasters. Students will explore how function properties affect compositions and how composition affects the graphs, including domain, of the functions. Finally, students will explore inverse relations. They will use mathematical and nonmathematical situations to develop an understanding of inverses. Students will analyze inverse relations to determine if they describe a one-to-one correspondence and learn to restrict the domain, if necessary, in order to define an inverse function. Students will verify inverse relations using symmetry in graphs and tables and using composition. Given a function, students will find the inverse function, if it exists, using algebraic techniques.
Polynomial and Rational Functions
Students will be able to: • Sketch and analyze graphs of quadratic and polynomial functions. • Use long division and synthetic substitution to divided polynomials. • Use the Fundamental Theorem of Algebra to determine the number of rational and real zeros. • Determine the domain, find asymptotes, and sketch rational functions. • Perform operations with rational expressions

• Know and apply the Binomial Theorem • Understand that rational expressions form a system analogues to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression. This unit builds on student's previous work with linear and quadratic functions to help students make sense of the behavior they see in a larger family of polynomial functions. Students will continue analyzing how polynomials model certain behaviors with differing rates of change, and how the degree of a polynomial relates to the number of real zeros. Students will also analyze end behavior and extrema. Students will apply this knowledge to choose appropriate models for scenarios based on the relationship between quantities. With regards to rational functions, students will analyze the behavior of rational functions using tables, graphs and real-world situations. Students will look at the relationship between average rates of change and rational functions, with particular attention to the rate of change as the function approaches an asymptote. Students learn about vertical, horizontal asymptotes. Limit notation is introduced and students analyze the limiting behavior of a function
Complex Numbers
Students will be able to: • Find the conjugate of a complex number • Use conjugates to find moduli and quotients of complex numbers • Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers); and explain why the rectangular and polar forms of a given complex number represent the same number. • Represent additional, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. • Extend polynomial identities to the complex numbers • Write equations in polar form, graph polar equations. This unit builds on students' previous work with complex numbers. Students will be introduced to the polar coordinate system. Students convert between the Cartesian and polar coordinate systems. They learn about basic polar graph families and explore the symmetry of graphs. Students also learn to express complex numbers in polar form and apply DeMoivre's Theorem to complex numbers.
Exponential and Logarithmic Functions
Students will be able to: • Recognize, evaluate, and graph exponential and logarithmic functions. • Rewrite logarithmic functions with different bases. • Use properties of Logs. • Solve exponential and logarithmic equations. • Use exponential growth, decay models. • Fit exponential and logarithmic models to data.

• Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. In this unit, students will build on and expand their understanding of exponential and logarithmic functions from Algebra II. Students will model a variety of scenarios using exponential or logarithmic functions, and will analyze those models using multiple representations such as tables and graphs. Rates of change will again be a focus, as students compare rates of change as the input values increase. Students will also study the inverse relationship between exponential functions and logarithmic functions, and will use this relationship to critically analyze problems.
Trigonometric Functions
Students will be able to: • Describe an angle and convert between degree and radian measures. • Identify a unit circle and its relationship to real numbers. • Evaluate trigonometric functions of any angle. • Use trigonometric identities. • Sketch graphs of trigonometric functions. • Compositions of trigonometric functions. • Model real life problems using trigonometric functions. • Understand that restricting a trig function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. In this unit, students will define the three foundational trigonometric functions: y = sin x, y = cos x and y = tan x. Students learn to transform these functions and will develop a stronger understanding of how parameters affects the period of the function. Students will develop an intuitive understanding of the unit circle, circular motion, and how trigonometric functions are used for modeling real-world problems. By the end of this unit, students will have a conceptual understanding of how trig functions are generated and will be able to use these functions to model various situations.
Analytic Trigonometry
Students will be able to: • Use fundamental identities to evaluate trigonometric functions. • Verify trigonometric identities. • Solve trigonometric equations using inverses. • Prove and use sum and difference formulas, product and sum formulas, multi-angle formulas, power reducing formulas, half-angle formulas. • Derive and Use Law of Sines and Cosines. • Derive the formula A = ½ ab sin(C) for the area of a triangle • Find areas of oblique triangles. In this unit, students will derive the trigonometry identities, including the Pythagorean identities, from the unit circles. Students will develop their logical thinking skills through verifying complex

identities. Students will then derive the sum and difference identities and the double/half identities. Emphasis will be on encouraging students to derive complex identities from simple, basic identities. Students will extend their work with right triangle trig from geometry to apply reciprocal trig functions in right triangle problem situations. They will then derive the Law of Sines and the Law of Cosines and use these laws to solve problems in various real-world scenarios. Students will also explore the case of the Law of Sines. Finally, students use trigonometry to find the area of triangles in a variety of ways, including Heron's formula.
Vector Analysis
Students will be able to: • Recognize vector quantities as having both magnitude and direction. • Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. • Solve problems involving velocity and other quantities that can be represented by vectors. • Perform operations with vectors • Find direction angles of vectors. • Find the dot product of two vectors. In this topic students learn how to represent a problem situation using vectors and will develop an understanding of magnitude, direction, and unit vectors. They will explore vector operations through geometry and resolution into components, including addition and subtraction, scalar multiplication, dot product, and cross product. Students will also find the angle between two vectors, and determine the vector equation of a line.
Matrices
Students will be able to: • Use matrices to represent and manipulate data • Perform operations with matrices • Understand that matrix multiplication for square matrices is not a commutative operation but still satisfies the associative and distribute properties. • Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. • Understand that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse • Work with 2x2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. • Multiply a vector by a matrix of suitable dimensions to produce another vector. • Represent a system of linear equations as a single matrix equation in a vector variable • Find the inverse of a matrix if it exists and use it to solve systems of linear equations.

Conic Sections and Analytic Geometry
Students will be able to: • Write the equations, analyze and sketch the graphs of parabolas, ellipses, and hyperbolas. • Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. • Use the discriminant and eccentricity to identify conics. • Solve systems of quadratic equations. • Rewrite sets of parametric equations and find sets of parametric equations for graphs. • Write equations of conics in polar form. • Rotate the coordinate axis to eliminate the x-y term in equations of conics and use the discriminant to identify conics. This unit first builds on previous work defining conic sections as the intersection of a cone and a plane, as well as locus definitions. In this topic, those ideas are used to model situations using the conic sections. The eccentricity of a conic section is defined and used to classify the conic sections. Finally, systems of conic sections are used to solve problems.
Sequences and Series
Students will be able to: • Use sequence, factorial, summation notation to write terms and sums of sequences. • Write arithmetic and geometric sequences. • Prove statements using induction. • Understand and use regression to model a scenario Students will further develop their understanding of infinite sequences and series. Students learn that an infinite amount of terms of a sequence can be added with the sum being a finite value. Students also learn that an infinite amount of polynomials can be added together and that the sum of these polynomials is an approximation for various functions such as e^x, sin(x), ln(x), and others. Students will also be introduced to mathematical induction and how it is used to prove some common identities in mathematics. In this unit, students use their knowledge of different parent functions, including linear, polynomial, power, exponential, logarithmic and trigonometric functions, to model a variety of situations based on data sets. Students will compare the fit of function models to various data and will use their knowledge of function characteristics to judge the validity of a model. They use regression and learn about correlation to determine the strength of a model. Students use the model to make predictions in the context of the situation given.
Statistics and Probability
Students will be able to:

- Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A) P(B|A) = P(B) P(A|B)
- Use permutations and combinations to compute probabilities of compound events and solve problems
- Define a random variable for quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding distribution using the same graphical displays as for data distribution.
- Calculate the expected value of a random variable and interpret it as a mean of the probability distribution.
- Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated.
- Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically.
- Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
- Use probabilities to make fair decisions
- Analyze decisions and strategies using probabilities concepts.

An Introduction to Calculus

Students will be able to:

- Estimate limits and use properties and operations of limits.
- Find limits by direct substitution and by using the dividing out and rationalizing techniques.
- Approximate slopes of tangent lines, use the limit definition of slope, and use derivatives to find slopes of graphs and to find max/min values
- Use Power Rule
- Derive and apply Intermediate Value Theorem
- Apply derivatives to real-world applications

In the final unit of the year, students will be introduced to calculus concepts, including limits and slopes of tangent lines. Students will apply their knowledge of end behavior, asymptotes, and removable discontinuities to develop an intuitive graphical understanding of limits. Once students have a firm conceptual understanding of limits, we will study algebraic techniques for solving for limits of polynomial and rational functions. Finally, students will be introduced to rates of change by using tangent lines to approximate rates of change.

Honors Final Exam Details

A comprehensive - 2 hour - final exam will be given at the end of each semester. The semester 1 final exam will cover units from Semester 1 and be comprised of multiple choice, short answer and performance task questions. The final at the end of semester 2 will encompass all units learned in Pre-Calculus Honors. The semester 2 final will also consist of multiple choice, short answer and performance task questions.

Students will also complete a project each semester. Projects include: instructional videos, power point / brochure, research papers, and computer simulation models.

Project example: Each group is assigned a different trigonometric modeling problem. They will prepare a presentation where they are expected to present a graph, an equation, a short explanation of each piece of its model and detailed solutions to the questions being posed. An example of one of the problems is the "Function Modeling Project": Height and Weight for Growing Children